ON (CA) TOPOLOGICAL GROUPS, II

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This note is a continuation of our study on CA groups [5]. We recall the definition of a CA group. Let G be a locally compact topological group. Let $A(G)$ be the collection of all bicontinuous automorphisms of G. Then $A(G)$ is a group with composition as the binary operation. Let $A(G)$ be topologized by the generalized compact-open topology. Then $A(G)$ is a topological group. We shall use $I(G)$ to denote the subgroup of $A(G)$ consisting of all the automorphisms induced by inner automorphisms of G. If $I(G)$ is a closed subgroup of $A(G)$, then G is called a CA group. For general properties of CA groups and previous results along these lines, refer to [8], [9], [3], [5], [6]. Most previous results on CA groups are concerning connected locally compact groups. Given a locally compact connected topological group G, by the structure theorem, there exists a compact normal subgroup K of G such that G/K is an analytic group. It is well known that $A(G/K)$ is a Lie group, *a fortiori*, the closure of $I(G/K)$ in $A(G/K)$ is an analytic group. Thus the theory of connected CA groups is very close to analytic groups. However, the situation becomes very complicated when G is not assumed to be connected. At the end of this note, we shall give some examples to show the complexity of the problems. In §2, we shall prove the following statement: Let G be an analytic group and H be a closed uniform subgroup of G. Assume H is a CA group, then G is also a CA group. (Previously D. H. Lee had proved this theorem for solvable groups.) This generalizes a result in [1]. In §3, we shall discuss the embedding of a CA group as a dense subgroup. A sufficient condition for absolute closedness is given, which generalizes previous results. A collection of examples are given in §4.

1. Let G be a σ-compact Lie group. Let $A(G)$ be the group of all bi-continuous automorphisms of G topologized by the compact-open topology [5]. Since G is locally connected, $A(G)$ is a topological group [2]. Moreover, $A(G)$ is closed in the group $H(G)$ of all homeomorphisms of the topological space G. Thus $A(G)$ is metrizable, and its topology is of second category [2]. Suppose G is also a CA group. Then $I(G)$ is closed in $H(G)$. This implies $I(G)$ is second category [2] and it is isomorphic with $G/Z(G)$, where $Z(G)$ denotes the center of G. Hence $I(G)$ is a Lie group, and we have the following statement:

1.1. Let G be a σ-compact CA Lie group. Then the group of all inner automorphisms under the compact-open topology is a Lie group.

1.2. *Notations and conventions.* Unless otherwise specified, all the groups

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