A SEMINORM TOPOLOGY FOR SOME (DF)-SPACES OF ENTIRE FUNCTIONS

By B. A. Taylor

1. Introduction. For $\Phi = \{\phi_1 \leq \phi_2 \leq \phi_3 \leq \cdots\}$ a family of functions on $\mathbf{C}^p$, let $E(\Phi) = \bigcup_n E(\phi_n)$, where $E(\phi_n) = \{f: f$ entire on $\mathbf{C}^p$ and $\|f\|_n = \sup |f(z)e^{-\phi_n(z)}| < +\infty\}$. Then each $E(\phi_n)$ is a Banach space and $E(\Phi)$ can be given a topology as the inductive limit (cf. [4; 223]) of the Banach spaces $E(\phi_n)$. Such spaces arise frequently as the dual spaces of Fréchet–Montel spaces (see e.g. [1], [2], [5], [6], [7], [8], [9]). However, for the study of some problems in these spaces, and particularly in determining if certain spaces are analytically uniform, in the sense of Ehrenpreis [1], [2], it is convenient to have a description of the topology of $E(\Phi)$ in terms of weighted seminorms. We show here, under some mild restrictions on the ϕ_n, that this can always be accomplished.

The seminorms to be considered are determined as follows.

Definition. $\mathcal{K} = \mathcal{K}(\Phi) = \{k(z)$: k is upper semicontinuous on C^p and $k(z) \geq \epsilon_n e^{\phi_n(z)}$ for some constants ϵ_n and all $n > 0$, $z \in C^p\}$.

For each $k \in \mathcal{K}$, $\|f\|_k = \sup\{|f(z)/k(z)|: z \in C^p\}$ is a seminorm on $E(\Phi)$, and this family of seminorms determines a locally convex topology on $E(\Phi)$.

Theorem 1. *Suppose that the ϕ_n satisfy the following conditions:*

(i) *each $\phi_n(z)$ is plurisubharmonic;*

(ii) $\phi_{n+1}(z) \geq \log(1 + |z|^2) + \tilde{\phi}_n(z)$, *where* $\tilde{\phi}_n(z) = \sup\{\phi(z + \xi): |\xi| \leq 1\}$ ($|\xi|^2 = \sum |\xi_i|^2$ *is the Euclidean norm on* C^p);

(iii) *for every $n > 1$, $\phi_{n+1}(z) - \phi_n(z)$ is bounded above on every bounded set. Then the topology determined on $E(\Phi)$ by the seminorms $\{\|\ \|_k: k \in \mathcal{K} = \mathcal{K}(\Phi)\}$ is the same as the topology given on $E(\Phi)$ as the inductive limit of the Banach spaces $E(\phi_n)$.*

We remark that in some special cases, the condition (ii) can be weakened to

(ii)′ $$\phi_{n+1}(z) \geq \tilde{\phi}_n(z).$$

The condition (ii)′ is needed only because we work with supremum norms instead of L^2-norms. The stronger condition (ii) is needed because we use the bound given by Hörmander in Theorem 4.4.2 of [3] for solutions to the equation $\bar{\partial}u = \omega$. In some special cases, Theorem 4.4.1 of [3] gives better results.

2. Proof of the theorem. From now on, we shall drop the Φ from our notation, and denote $E(\Phi)$ by just E. When this vector space is equipped with the

Received June 12, 1969. This paper represents results obtained at the Courant Institute of Mathematical Sciences, New York University, under the sponsorship of the Sloan Foundation.