DECOMPOSTIONS OF E^3 AND THE TAMENESS OF THEIR SETS OF NON-DEGENERATE ELEMENTS

BY JOHN P. RILEY, JR.

I. Introduction. This paper investigates the properties of certain upper semi-continuous decompositions of E^3 whose non-degenerate elements form a compact 0-dimensional subset of the decomposition space. Armentrout [2] has shown that if C is a compact 0-dimensional subset of E^3, then there exists a point-like upper semi-continuous decomposition of E^3 such that there is a homeomorphism of the decomposition space onto E^3 taking the image of the non-degenerate elements homeomorphically onto C. This paper gives some partial results about the relation of the tameness of the compact 0-dimensional set to the type of non-degenerate elements and the way that they are embedded in E^3. In particular, there are some results concerning decompositions whose non-degenerate elements are straight line segments.

Most of the terminology used in this paper is standard. The reader is referred to [1], [3], [4], and [7].

If p is a point in E^3 and ϵ is a positive number, then by $N(p, \epsilon)$ we will mean the collection of all points of E^3 whose distance from p is less than ϵ.

If S is a 2-sphere in E^3, then by Int S we will mean the bounded component of $E^3 - S$ and by Ext S, the unbounded component.

II. Tameness and decomposition spaces. In this section we will make use of several theorems from Bing [3], modified for compact 0-dimensional sets rather than Cantor sets. Starting with a compact 0-dimensional set K in E^3 satisfying the hypotheses of Bing's Corollary 3.2, we may carefully add points if necessary to get a Cantor set which is tame, from which we may conclude that K is tame. It follows readily from this result that Bing's Theorems 3.1, 4.1, 4.2, and 4.3 hold for compact 0-dimensional sets.

The following lemma will also be necessary in this section.

LEMMA A. *Let H be a closed set in E^3 and let h be a component of H. Let ϵ be a positive number and let S_0 be a polyhedral 2-sphere such that $h \subset$ Int S_0 and $S_0 \cup$ Int $S_0 \subset N(h, \epsilon)$. Suppose that for each component g of H which intersects S_0, there is a polyhedral 2-sphere $S(g)$ such that $g \subset$ Int $S(g)$, $h \subset$ Ext $S(g)$, $S(g) \cup$ Int $S(g) \subset N(h, \epsilon)$, and $S(g) \cap H = \phi$. Then there is a polyhedral 2-sphere S such that $h \subset$ Int S, $S \subset N(h, \epsilon)$, and $H \cap S = \phi$.*

Proof. This proof of this lemma uses standard techniques developed by Bing and will be omitted.

Let G be an upper semi-continuous decomposition of E^3, and let K be a

Received April 23, 1969; in revised form August 21, 1970.