A NON-METRIC INDECOMPOSABLE CONTINUUM

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Let $A = [1, \infty)$. We wish to consider $A^* = \beta(A) - A$, where $\beta(A)$ denotes the Stone-Čech compactification of A. If X is a compact Hausdorff space and $f: A \to X$ a continuous map, f^* will denote the unique extension of f to $\beta(A)$. The principal result follows:

THEOREM 1. *A^* is an indecomposable continuum.*

Proof. To prove that A^* is a continuum, let $A_n = [n, \infty)$ for each n. Then

$$A^* = \bigcap_{n=1}^{\infty} (A_n \cup A^*)$$

$$= \bigcap_{n=1}^{\infty} \overline{A_n}$$

and since $\overline{A_n}$ is a compact continuum for each n, so is A^*.

To show that A^* is indecomposable, suppose X, Y are proper closed subsets of A^* and $A^* = X \cup Y$. It suffices to show that X is not connected. To this end, let $x \in A^* - Y$, $y \in A^* - X$. Since $\beta(A)$ is regular, there exist sets U, V open in $\beta(A)$ such that:

1) $x \in U$ and $y \in V$.
2) $\bar{U} \cap \bar{V} = \bar{U} \cap Y = \bar{V} \cap X = \phi$.

Observe that $U \cap A$ and $V \cap A$ are unbounded. Choose sequences $\langle p_i \rangle_{i=1}^{\infty}$, $\langle q_i \rangle_{i=1}^{\infty}$ and $\langle r_i \rangle_{i=1}^{\infty}$ from A as follows:

Let $p_1 \in U \cap A$, then choose $q_1 > p_1$ such that $q_1 \in V$. This is possible since $V \cap A$ is unbounded. Then choose $r_1 > q_1$ such that $(q_1, r_1) \subseteq V$. This is possible since V is open and hence q_1 lies in some open interval in V.

Proceeding inductively, suppose p_k, q_k, and r_k have been chosen for $k < n$ such that for each k:

1) $p_k \in U$.
2) The interval (q_k, r_k) is contained in V.
3) $p_k < q_k < r_k$, and if $k < n - 1$, then $r_k < p_{k+1}$.

Then since $U \cap A$ is unbounded, it is possible to choose $p_n > r_{n-1}$ such that $p_n \in U$. Since $V \cap A$ is unbounded, there exists a $q_n > p_n$ such that $q_n \in V$. Since V is open, r_n may be chosen greater than q_n such that $(q_n, r_n) \subseteq V$.

Then the sequences $\langle p_i \rangle$, $\langle q_i \rangle$, and $\langle r_i \rangle$ are unbounded, for if not they have a

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