EQUIVARIANT BORDISM AND $(Z_2)^k$ ACTIONS

By R. E. Stong

1. Introduction. The object of this paper is the analysis of the bordism classification of pairs (M, φ) where M is a closed differentiable manifold and φ: $(Z_2)^k \times M \rightarrow M$ is a differentiable action of the group $(Z_2)^k = Z_2 \times \cdots \times Z_2$ (k copies) on M. In their monumental work [2], introducing bordism methods to the study of group actions, Conner and Floyd made a fairly complete analysis of the case $k = 1$ (Chapter IV) and established a few results in the general case (Chapter V).

In a later work [3], Conner and Floyd established methods for bordism analysis of G actions making use of decompositions of G through group extensions. Their main application was to $G = Z_{p^r}$, with p an odd prime, but their methods are clearly suitable for the study of $(Z_2)^k$ actions. As always in the application of general methods, there are a number of details to be studied more closely. In particular, special properties of $(Z_2)^k$ give far more complete results concerning stationary point structure. (*Note*: $x \in M$ is a stationary point of the $(Z_2)^k$ action if $\varphi(t, x) = x$ for all $t \in (Z_2)^k$.)

The main result of this paper is basically an extension of Conner and Floyd's theorem [2, 30.1] to:

Proposition. *If* φ: $(Z_2)^k \times M \rightarrow M$ *is a differentiable action without stationary points on a closed manifold, then* (M, φ) *bounds as a manifold with* $(Z_2)^k$ *action; i.e. there is a differentiable action* ψ: $(Z_2)^k \times V \rightarrow V$ *on a compact manifold with boundary such that* $\partial V = M$ *and* ψ *restricts to* φ *on* ∂V.

(*Note*: Under these hypotheses, (30.1) says that M bounds as a manifold.)

This strengthening of the Conner and Floyd theorem implies that the stationary point structure completely determines the bordism class of a $(Z_2)^k$ action.

Peripherally, this paper will also consider the equivariant bordism groups given by equivariant maps f: $(M, \varphi) \rightarrow (BO(n), \tau_n)$ with τ_n an appropriate action of $(Z_2)^k$ on $BO(n)$, or equivalently, the bordism classification of n-plane bundles with $(Z_2)^k$ acting by bundle maps and covering a $(Z_2)^k$ action on a closed manifold. Again, the main result is that stationary point structure determines the bordism class.

2. $(\mathfrak{F}, \mathfrak{F}')$ - free actions. To begin the analysis of $(Z_2)^k$ actions, it is desirable to recall the definitions and results of Conner and Floyd [3]. A preliminary and less detailed analysis in the unoriented case, as needed here, appears in Conner [1].

Received December 20, 1968. The author is indebted to Princeton University and the NSF for financial support during this work.