LOCAL PEAK SETS OF UNIFORM ALGEBRAS

BY WILLIAM E. MEYERS

1. Introduction. By a *uniform algebra* on a topological space X we shall mean an algebra A of continuous complex-valued functions on X which contains the constants and is closed under uniform convergence on compact subsets of X. For example, the algebra Hol (X) of functions analytic on an open set X in C^n is a uniform algebra on X, and if A is a function algebra (a point-separating sup norm closed subalgebra, containing the constant functions, of continuous functions on a compact space X) then A is a uniform algebra on X. Rossi has shown that if A is a function algebra on X and $X = M(A)$, the space of non-zero continuous homomorphisms of A into C with the weak* topology, then every local peak set of A in X is a global peak set. This is not true in general for uniform algebras [2], but in this paper the following result is established: if S is a compact local peak set of a uniform algebra A on a space X, if A is an F-algebra with $M(A) = X$, and if K is any compact subset of X containing S, then S is a peak set of the restriction algebra $A \mid K$ (not closed!).

Using this result, it is possible to show that every compact local peak set contains a local peak point when $M(A)$ is first countable, and to obtain a characterization of uniform algebras having no local peak points in $M(A)$. When A is a uniform algebra on an open subset X of the complex plane C with $M(A) = X$ and such that the function $z(x) = x$ lies in A, this characterization, together with a result of Rudin, yields that A has no local peak points if and only if $A =$ Hol (X). Finally, an example is given which shows that a non-compact local peak set need not contain a local peak point.

In order to prove the main result about local peak sets, we use an extension of a well-known result concerning hulls of ideals in Banach algebras. For a subset J of a commutative locally m-convex topological algebra A, define the *hull* of J to be the set $\{x \in M(A) : x\ (a) = 0,\ a \in J\}$. Thus the hull of J is the intersection of zero sets of the Gelfand transforms $\hat{a}$, $a \in J$, where $\hat{a}$ is defined by $\hat{a}(x) = x(a)$, $x \in M(A)$. If A is a commutative Banach algebra, then it is known that every compact open and closed subset of a hull is a hull (cf. [4; 169] or [5; 10]). In this paper this result is extended to commutative F-algebras, since it is in this context that we shall need to use it.

2. Preliminaries. Let A be a complete locally m-convex topological algebra and let $M(A)$ be the space of continuous homomorphisms of A into C with the weak* topology. A may be written as the inverse limit of a system $\{A_\alpha, \pi_\alpha^\beta\}$ of Banach algebras and maps $\pi_\alpha^\beta : A_\alpha \rightarrow A_\beta$, $\alpha \geq \beta$. Since the dual maps to

Received November 18, 1968.