ON THE LARGEST PRIME DIVISORS OF IDEALS IN FIELDS OF DEGREE n

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1. In 1947, Chowla and Vijayaraghavan [3] investigated the number of rational integers not exceeding x and having at least one prime factor greater than x, where θ is a fixed number greater than 1. In 1949, Buchstab [1] considered the problem of integers in an arithmetic progression all prime factors of which are small in magnitude. From [3] and [1], if $\Phi_1(x, x^{1/\theta})$ counts the number of rational integers not exceeding x which have no prime factors greater than $x^{1/\theta}$, we have

$$\Phi_1(x, x^{1/\theta}) = xW(\theta) + O\left(\frac{x}{\log x}\right),$$

where $W(\theta)$ is the continuous, positive and decreasing function defined by

$$W(\theta) = 0 \quad \text{for} \quad \theta < 0, \qquad W(\theta) = 1 \quad \text{for} \quad 0 \leq \theta \leq 1;$$

$$W(\theta) = 1 - \log \theta \quad \text{for} \quad 1 \leq \theta \leq 2 \tag{1}$$

and

$$\theta W'(\theta) = -W(\theta - 1) \quad \text{for} \quad \theta \geq 2.$$

de Bruijn [5] has shown that

$$\begin{aligned} W(\theta) &= \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \exp\left\{-\theta v + \gamma + \int_0^v \frac{e^t - 1}{t}\right\} dv \\ &= \exp\{-\theta \log \theta - \theta \log \log \theta + O(\theta)\}, \qquad \theta > 3, \end{aligned}$$

where γ is Euler's constant. In 1950, de Bruijn [6] improved on the results for $\Phi_1(x, x^{1/\theta})$ and obtained

$$\Phi_1(x, x^{1/\theta}) = \Lambda(x, x^{1/\theta}) + O\left(x\theta^2 \exp\left\{-c\left(\frac{\log x}{\theta}\right)^{\frac{1}{2}}\right\}\right),$$

where c is a constant and

$$\Lambda(x, x^{1/\theta}) = x \int_0^\infty W\left(\frac{\theta(\log x - \log t)}{\log x}\right) \frac{d[t]}{t}.$$

More recently, Jordan [7], has shown that

$$\underset{x \to \infty}{Lt} \frac{\Omega(x, x^{1/\theta})}{\pi x} = 1 - W(\theta)$$

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