CONCERNING THE SEPARATION OF CERTAIN PLANE-LIKE SPACES BY COMPACT DENDRONS

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Suppose S is a complete Moore space (which includes the class of complete metric spaces) which is connected, locally connected, has no cut point and such that the Jordan curve theorem is true. That is, suppose R. L. Moore's Axioms 0, 1-4 of [4] are satisfied. (Extensive use is made of [4]. Theorems in this book will be referred to as *Foundations*, followed by the number in Roman numerals of the chapter in which it occurs, followed by the number of the theorem in Arabic numerals. Thus, *Foundations* IV, 9 refers to Theorem 9 of Chapter IV. Much of the terminology is that of this book.) Spaces satisfying these axioms have many properties of the plane. Many theorems of the plane related to intersecting simple closed curves and arcs can be proved on this basis and will be used in this paper, some without specific mention. Nonetheless, as Moore has pointed out, these spaces need not be metric, separable or locally compact. Indeed, most of the results of this paper concern spaces which differ from the plane in an important way. In particular, most of the spaces considered here have endpoints.

In a 1941 paper [1], F. B. Jones investigated the separation of spaces satisfying Axioms 0, 1-4 by arcs. Four of his results are stated below and will be used. Theorems 1 and 2 will be extended to compact dendrons.

Theorem 1. *If the arc AB separates S, then A or B is an endpoint of S.*

Theorem 2. *Suppose AB is an arc which separates S. (1) If only one of the points A and B is an endpoint of S, then $S - AB$ has no more than two components. (2) If both A and B are endpoints of S, then $S - AB$ has no more than three components.*

Theorem 3. *If the arc AB separates S, then some interval of AB is irreducible with respect to the property of containing A and separating S.*

Theorem 4. *There do not exist three mutually exclusive connected domains whose boundaries have a common free segment.* (This is a modification of Jones' incorrectly stated "Theorem" 4 of [1] that no arc lies in the boundary of each of three mutually exclusive connected domains and is established by his argument.)

Definitions. The point E of the continuous curve M is said to be an endpoint of M provided E is an endpoint of every arc in M containing E. A dendron

Received October 18, 1968. Presented to the American Mathematical Society April 20, 1968.