ON SINGLY-GENERATED LOCALLY M-CONVEX ALGEBRAS

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1. Introduction. In [1] R. Arens gave a condition on a singly-rationally-generated F-algebra A with identity which insured that A was the direct sum of its radical and an algebra $A(D)$, the algebra of all analytic functions on an open subset D of the plane. In [2] F. Birtel gave a condition which would characterize the entire functions in the class of singly-generated Liouville F-algebras with identity. Arens' condition was the existence of a continuous derivation which satisfied Cauchy estimates with respect to some sufficient family of seminorms on the F-algebra. Birtel's condition was on the spectra of the images of the generator in each of a sequence of Banach algebras whose inverse limit was the algebra in question.

Our main results are the same type, this note having its roots in a study of Birtel's paper. We show, for example, that if A is a singly-generated (singly-rationally-generated) commutative, complete LMC algebra with identity and if the type II boundary $\partial_2(A)$ introduced in [4] is empty, then A is the direct sum of its radical and an algebra $A(D)$ where D is an open, polynomially convex subset of the plane (open subset of the plane) homeomorphic to the spectrum of A. We also consider a singly-generated, commutative, complete LMC algebra with identity $A = \lim_\alpha \operatorname{inv} A_\alpha$ and assume that for some generator x $\lim_\alpha \inf \operatorname{bdy}_{\sigma(x)} \sigma_\alpha(x)$ is empty. In this case $x\hat{}$ is a topological map, $\partial_2(A) = \sigma(x) \cap \operatorname{bdy} \sigma(x)$ and the compact-open completion of $\hat{A} \mid (M - \partial_2(A))$ is $A(\operatorname{int} \sigma(x))$.

The results of §3 are related to the problem considered by Birtel in [2] of constructing an F-algebra from algebras of continuous functions on an ascending sequence of compact subsets of the plane. We obtain in a more general setting relations between various conditions on the relationships between the compact sets and their union and the continuity of the functions in the algebra generated.

Finally, we close §4 by correcting the statement of *Condition* Φ in [2] to agree with its use there and by showing that Theorem 3.3 of that paper then follows from our Theorem 4.3.

2. Preliminaries. A *locally m-convex* (LMC) algebra ia a locally convex (Hausdorff) topological algebra whose topology is generated by a family of seminorms (submultiplicative, convex, symmetric functionals). A *kLMC* algebra is one in which there exists a defining family of seminorms each of which satisfies $||x^2|| = ||x||^2$ for every x in the algebra. We let $(A, \{||\cdot||_\alpha : \alpha \in \mathfrak{A}\})$ be an LMC algebra, where we may assume that $\mathfrak{A}$ is directed and that if $\alpha \leq \beta$, then $||x||_\alpha \leq ||x||_\beta$ for each $x \in A$. Each seminorm generates a Banach algebra A_α,

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