ON CA TOPOLOGICAL GROUPS

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The purpose of this note is a topological investigation into a certain class of locally compact groups, which are known as CA groups. A locally compact group G is called a CA group if the group of inner automorphisms of G is closed in the group of automorphisms of G. This concept has been introduced in Van Est [4], [5] and Omori [3] has recently obtained some of the important properties of these groups. As is well known, every semi-simple analytic group is CA and compact groups are easily seen to be CA. Another important class of CA groups is that of all nilpotent Lie groups. In the case of solvable groups, the situation seems less favorable, as there is a five-dimensional simply connected solvable non-CA group (see §3) whose algebraic structure is rather simple. Indeed, this example motivated us to investigate structures of non-CA groups. The first section contains some of the preparatory materials for the latter use. In §2, we extend a result of Omori [3] on CA Lie group to any locally compact CA group. The last section is devoted for a characterization of non-CA solvable analytic groups and some of consequences of this result.

Throughout this note a morphism of topological groups refers to a continuous homomorphism of topological groups, and the words automorphisms, isomorphisms are used similarly. For a subset M of a topological group G, $\bar{M}$ denotes the closure of M in G, and the identity component of a topological group G is denoted by G_0. Finally $Z(G)$ for a group G denotes the center of G.

1. Locally compact CA groups.

1.1 Let G be a locally compact group and Aut (G) the group of all automorphisms of G. There is a natural topology in Aut (G) under which Aut (G) becomes a topological group. This topology is known as the generalized compact open topology. If G is an analytic group, this topology coincides with the topology of Aut (G) as a linear Lie group over the Lie algebra of G. Let Aut_0 (G) denote the connected component of the identity of Aut (G), and Int (G) the subgroup of Aut (G) consisting of all inner automorphisms of G. More generally, the following notation is adopted throughout the remainder of this paper. For a subset A of G, $\mathrm{Int}_G(A)$ denotes the subset of inner automorphisms which are induced by elements in A.

1.2 We present here a structure theorem of connected locally compact group for later use. Let G be a connected locally compact group. Then G is locally

Received October 4, 1968. The second author was partially supported by NSF GP-7527 and NSF g-GP-8961.