ON UNIFORM DISTRIBUTION OF SEQUENCES IN $GF[q, x]$ AND $GF\{q, x\}$.

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1. Introduction and preliminaries. Let $\Phi = GF[q, x]$ denote the ring of polynomials in the indeterminate x over an arbitrary finite field $GF(q)$ of q elements. If A and M are any two elements of Φ with $\deg M > 0$, let $A(M)$ be the uniquely determined element of Φ such that $\deg A(M) < \deg M$ and $A \equiv A(M) \pmod M$.

J. H. Hodges [2; 55] defined the uniform distribution of a sequence $\theta = (A_i)$ of elements of Φ as follows. Let M be any element of Φ with $\deg M = m > 0$. For any $B \varepsilon \Phi$ and integer $n \geq 1$ define $\theta(n, B, M)$ as the number of terms among $A_1, A_2, \cdots, A_n$ such that $A_i(M) = B(M)$. Then the sequence θ is said to be *uniformly distributed modulo M in Φ* if

$$\lim_{n\to\infty} n^{-1}\theta(n, B, M) = q^{-m} \quad \text{for all} \quad B \varepsilon \Phi. \tag{1.1}$$

The sequence θ is said to be *uniformly distributed in Φ* if (1.1) holds for every $M \varepsilon \Phi$ with $\deg M = m > 0$.

Let $\Phi' = GF\{q, x\}$ denote the extension field of Φ consisting of all the expressions

$$\alpha = \sum_{i=-\infty}^{m} c_i x^i \qquad (c_i \varepsilon GF(q)).$$

If α has this representation and $c_m \neq 0$, then we define $\deg \alpha = m$. We extend this definition by writing $\deg 0 = -\infty$. The integral and fractional parts of α, denoted by $[\alpha]$ and $((\alpha))$ respectively, are defined by

$$[\alpha] = \sum_{i=0}^{m} c_i x^i, \qquad ((\alpha)) = \sum_{i=-\infty}^{-1} c_i x^i.$$

It follows from the definition, that, for α and β in Φ', we have $[\alpha + \beta] = [\alpha] + [\beta]$. We say $\alpha \equiv \beta \pmod 1$ if $\alpha = \beta + A$, where $A \varepsilon \Phi$. It follows that $\alpha \varepsilon \Phi'$ is congruent modulo 1 to a unique β, namely $\beta = ((\alpha))$, such that $\deg \beta < 0$.

L. Carlitz [1; 190] defined the uniform distribution of a sequence $\theta = (\alpha_i)$ of elements of Φ' in the following way. For any $\beta \varepsilon \Phi'$ and any positive integers n and k, define $\theta_k(n, \beta)$ as the number of terms among $\alpha_1, \alpha_2, \cdots, \alpha_n$ such that $\deg ((\alpha_i - \beta)) < -k$. Then the sequence θ is *uniformly distributed modulo 1 in Φ'* if

$$\lim_{n\to\infty} n^{-1}\theta_k(n, \beta) = q^{-k} \quad \text{for all} \quad k \quad \text{and} \quad \beta \varepsilon \Phi'. \tag{1.2}$$

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