DESARGUESIAN DECOMPOSITIONS FOR PLANES OF ORDER p^2

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I. Introduction. In [4] the author attemped to show that a number of constructions of finite planes, including the construction called derivation [2], can be interpreted as special cases of what the author calls net replacement.

A basic idea underlying this approach is to identify the points of an affine plane (of prime power order) with the points of a Desarguesian plane of the same order. It then becomes natural to attempt to represent the collineations in terms of operations in a field. In [5] the author considered affine planes which contain nets such that the lines of the nets in question can also be identified with the lines of a Desarguesian plane. A plane can contain several nets of this type. Under suitable restrictions, these "Desarguesian nets" act as sets of imprimitivity for the collineation group when it is considered as a permutation group on the lines. Again, under suitable restrictions, the stabilizer of one of these nets is a subgroup of the collineation group of a Desarguesian plane. In a large class of cases, the full collineation group is a subgroup of the collineation group of a Desarguesian plane. In most specific instances to which this kind of analysis can be applied, the full collineation group can be obtained explicitly.

However, certain difficulties arise for planes of square order. In this paper we look at planes whose order is the square of an odd prime. In particular, we are interested in translation planes which can be constructed from Desarguesian planes by replacement of disjoint nets in the case where each net replacement is essentially a derivation. The André planes of order p^2 belong to this class. We show that this class contains planes which are not André planes. Our main result is that the full collineation group is a subgroup of the collineation group of a Desarguesian plane of the same order. That is, our results are of the same nature as those in [5] but apply to cases where the hypotheses of the Theorems in [5] are not satisfied.

§II contains some further preliminary discussion, together with some of the definitions and theorems from [5].

§III contains some background work on derivable nets embedded in planes.

In §IV we begin the analysis of ways of choosing disjoint derivable nets.

§V contains our main results.

In §VI we show some ways of choosing disjoint derivable nets which lead to planes which are not André planes.

II. Basic theorems and definitions. It follows from the work of André [1] that a translation plane of order p^r may be interpreted as follows: We have a vector space of dimension $2r$ over $GF(p)$ and a class of $p^r + 1$ mutually inde-

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