ON BAER AND QUASI-BAER RINGS

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Introduction. Kaplansky [6] introduced the Baer rings as rings in which every left (right) annihilator ideal is generated by an idempotent.

The motivation comes from the observation that the theory of rings of operators on a Hilbert space is a particular case of a pure algebraic theory of Baer rings satisfying some axioms.

Clark [2] introduced the quasi-Baer rings as rings in which every two-sided annihilator ideal is generated by an idempotent.

In §2 we study properties of quasi-Baer rings that lead to the structure of Artinian quasi-Baer rings.

In §3 we discuss Baer rings and we consider also the following conjectures of Clark for Artinian Baer rings:

(i) If the basic subring of an Artinian ring R is Baer, then R is Baer.
(ii) R is a Baer ring iff: (a) $xey = 0$ iff $xe = 0$ or $ey = 0$ for every primitive idempotent e in R and (b) R surrounds no zeros.

We furnish a counter example to (i). Under the restriction on R that fRg is a one-dimensional left vector space over fRf, for every pair of primitive idempotents f, g in R, we prove that (ii) implies that R is an hereditary ring, whence a Baer ring.

In §4 we discuss some relations between semi-primary quasi-Baer rings and their homological dimension.

In §5 the tensor product of certain quasi-Baer rings is proved to be a quasi-Baer ring.

1. Preliminaries. We assume that every ring has an identity, every ideal is a left ideal and every module over a ring R is a unitary left R-module, unless otherwise specified.

Let R be a ring. We denote: by N the Jacobson radical of R, by R_n the $n \times n$ matrix ring over R, by $T_n(R)$ the ring of (lower) triangular $n \times n$ matrices over $R(n > 1)$, by E_{ij} the $n \times n$ unit-matrix whose $(km)^{-\text{th}}$ component is $\delta_{ki}\delta_{mj}$. We refer to [7] for the definitions of the complete decomposition of a ring, the basic ring and related topics and to [3] for the homological concepts.

A two-sided ideal I in R is a prime ideal iff $x, y \in R$ and $x, y \notin I$ imply that $xRy \not\subset I$.

A ring R is a prime ring iff (0) is a prime ideal in R.

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