DIVISIBILITY OF THE GROUP OF DIVISOR CLASSES OF DEGREE ZERO OF A FUNCTION FIELD OF GENUS ONE

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1. Introduction. It is well known that the Jacobian variety of a complete, non-singular variety of dimension one (an algebraic curve) is a divisible group. This divisibility is a consequence of the structure properties of abelian varieties.

The divisibility of the Jacobian variety plays a role in class field theory through the isomorphism of the group of divisor classes of degree zero of the function field of an algebraic curve to the Jacobian variety of that curve. Namely, in inquiring whether a reciprocity law with the idele class groups as a system of groups can be defined for an algebraic function field in one variable having a non-finite quasi-finite field as field of constants, Rim and Whaples employ this divisibility in [5]. This being the case, a direct algebraic proof of this divisibility might be of interest. Such a proof is offered here for the case of a function field of genus one.

Special attention will be given to the characteristic two case which, it seems, has been somewhat neglected in the literature.

2. Preliminaries. Throughout, we assume that F is an algebraically closed field which serves as the field of constants for an algebraic function field K of one variable and genus one.

For a field k with $F \subsetneq k \subset K$, $P(k)$ denotes the set of all k-places which are identity on F. The group of k-divisors is denoted by $D(k)$ and is the free abelian group generated by $P(k)$. The group of k-divisor classes is the quotient group of $D(k)$ by the subgroup of principal k-divisors (the multiplicative group of k is embedded in $D(k)$ as usual, and (α) denotes the divisor of $\alpha \in k^*$). For $A \in D(k)$, $d(A)$ denotes the degree of A. If $A = \sum_{p \in P(k)} n_p p$ and $B = \sum_{p \in P(k)} m_p p$ are two k-divisors, we agree that $A \geq B$ if and only if $n_p \geq m_p$ for all $p \in P(k)$, and then we let $L(A) = \{\alpha \in k : (\alpha) \geq -A\}$. We regard $(0) > A$ for all A.

$L(A)$ is a finite dimensional vector space over F. Denoting this dimension by $l(A)$, the Riemann-Roch Theorem states that there is a non-negative integer g, referred to as the genus of k, such that for any $A \in D(k)$,

$$l(A) = d(A) + 1 - g + l(C - A)$$

where C is a k-divisor representing the so called "canonical class."

One easily sees that $l(C) = g$, $d(C) = 2g - 2$, and that if $d(A) > 2g - 2$, then

$$l(A) = d(A) + 1 - g.$$

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