# A PAIR OF NON-INVERTIBLE LINKS

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An oriented, ordered link $L$ of $\mu$ components tamely imbedded in the oriented 3-sphere $S$ will be called *invertible* if and only if there is an orientation-preserving autohomeomorphism of $S$ which takes each component of $L$ onto itself with reversal of orientation. While the existence of non-invertible knots [4] guarantees non-invertible links, it is of interest to have examples of non-invertible links with components all of which are invertible. It is the purpose of this paper to prove that the links pictured in Figures 1 and 2 are, in fact, non-invertible.

*Remark* 1. Each component of both links belongs to the knot type of $5_1$ and is, therefore, invertible. The link of Fig. 2 is obviously interchangeable. A glance at the (normalized) Alexander polynomial

$$\begin{aligned}\Delta(x, y) = {} & x^6(y^4 - 2y^3 + 2y^2 - y) + x^5(-2y^4 + 3y^3 - 3y^2 + 2y - 1) \\ & + (x^4 - x^3 + x^2)(y^4 - y^3 + y^2 - y + 1) \\ & + x(-y^4 + 2y^3 - 3y^2 + 3y - 2) + (-y^3 + 2y^2 - 2y + 1)\end{aligned}$$

of the link of Fig. 1 shows that it is not an associate of any of the four $L$-polynomials, $\Delta(y^{\epsilon_1}, x^{\epsilon_2})$, where each of $\epsilon_1$ and $\epsilon_2$ is either $+1$ or $-1$. Hence, the link of Fig. 1 cannot be interchanged.

*Remark* 2. Let $L = +K_1 \cup \cdots \cup + K_\mu$ be a link of $\mu$ components, $S_\mu$ the symmetric group of degree $\mu$, and $Z_2^{\mu+1}$ the direct product of $\mu + 1$ copies of the multiplicative group $Z_2 = \{-1, 1\}$. Define the *link-symmetric group* $\Gamma_\mu$ of degree $\mu$ (see [5]) to be the split extension of $Z_2^{\mu+1}$ by $S_\mu$, where $S_\mu$ permutes the last $\mu$ factors of $Z_2^{\mu+1}$. We shall say that $L$ *admits* $\gamma = (\epsilon_0, \epsilon_1, \cdots, \epsilon_\mu, p)$ of $\Gamma_\mu$, where $\epsilon_i = \pm 1$ and $p$ belongs to $S_\mu$, provided there is an autohomeomorphism $\psi$ of $S$ such that $\psi(+S) = \epsilon_0 S$, and $\psi(+K_\alpha) = \epsilon_\alpha K_{p(\alpha)}$ for each $\alpha$. To say that $L$ is invertible, then, is to say that $L$ admits $\gamma = (1, -1, \cdots, -1, (1))$ belonging to $\Gamma_\mu$.

Let $K$ be an oriented knot in $S$ and $G = \pi_1(S - K)$. Following Trotter [4], we shall call an element of $G$, which has linking number $+1$ with $K$, a *meridian* of $K$ provided that for any neighborhood $N$ of $K$ the element can be represented by a path $\gamma\beta\gamma^{-1}$, where $\gamma$ runs from the basepoint to a point of $N - K$, and $\beta$ is a loop in $N - K$ such that $\beta \sim 0$ in $N$. If, however, for any neighborhood $N$ of $K$ an element of $G$ can be represented by a path $\gamma\beta\gamma^{-1}$, where $\beta$ is a loop in $N - K$ such that $\beta \sim K$ in $N$ and $\beta \sim 0$ in $S - K$, the element is called a *longitude* of $K$. Any automorphism of $G$ taking the class of meridians into the

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