THE PRODUCT OF AN n-CELL MODULO AN ARC IN ITS BOUNDARY AND A 1-CELL IS AN $(n+1)$-CELL

By Peter W. Harley

1. Introduction and statement of main results. Each k-dimensional factor of I^n is a cell provided that $k \leq 2$ or $n \leq 4$. This fact is due to a combination or results found in papers of Bing [1], Szumbarski [17], and Young [19]. Bing states in [1] that as a consequence of Theorem 1 of [19], if one factor of I^n is $(n - 1)$-dimensional, then the other is an arc. Hence an $(n - 1)$-dimensional factor of I^n is a space X such that $X \times I \approx I^n$. The problem that motivated this paper was that of finding uncountably many solutions to the above equation, for $n > 4$. Having considered this problem we are led to the natural counterpart for cells of the well-known theorem of Curtis and Andrews [6] (which states that the product of E^n modulo an arc and E^1 is E^{n+1}) for Euclidean spaces. Thus we have established the following:

Theorem 1. *If A is an arc in the boundary of the standard n-cell I^n, then $I^n/A \times I \approx I^{n+1}$.*

As a corollary we see that there are at least as many factors of I^n, $n > 4$, as there are arcs in S^{n-2} with non-homeomorphic complements. It is conjectured that there are uncountably many such arcs, but it is not known if there are infinitely many. Three are given for S^3 in Artin-Fox [7].

To illustrate another application of Theorem 1, we give an example of a non-manifold X (cf. Glaser [8]) such that $X \times I \approx I^5$ and X can be written either as (1) the union of two 4-cells meeting in a 3-cell or as (2) the union of two 4-cells meeting in a 4-cell.

Poenaru [16] and Mazur [13] have given examples of combinatorial 4-manifolds different from I^4 whose products with I are cells, and Curtis [5] and Glaser [9] have given similar examples for $n \geq 4$. These methods are not directly applicable to the problem above, since there are only countably many compact combinatorial manifolds. Kwun and Raymond [11] have shown that the product of I^n modulo an arc in its interior and I^2 is I^{n+2}. This technique will produce non-manifold $(n - 1)$-dimensional factors of I^n, but in general the factors are are not readily distinguishable.

The proof of Theorem 1 is long and involves the work of McCauley [14], Connell [4], and Curtis and Andrews [6] as well as some more recent developments

Received April 5, 1967. This research constitutes a portion of the author's Ph.D. dissertation at the University of Georgia under the supervision of Professor J. C. Cantrell, to whom the author would like to express his appreciation. It was presented to the AMS on January 26, 1967 at Houston, Texas.