REFLECTION LAWS OF HIGH ORDER ELLIPTIC DIFFERENTIAL EQUATIONS IN TWO INDEPENDENT VARIABLES WITH CONSTANT COEFFICIENTS AND UNEQUAL CHARACTERISTICS ACROSS ANALYTIC BOUNDARY CONDITIONS

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1. Introduction. The purpose of this paper is to study the reflection laws of solutions of elliptic differential equations of the form

$$(1.1) \qquad L[u] = \sum_{k+j=2n} a_{kj} \frac{\partial^{2n} u}{\partial x^k \, \partial y^j} = 0, \qquad a_{kj} \text{ real constants}$$

all of whose characteristics are distinct, across an analytic arc κ on which the solution satisfies n analytic linear differential boundary conditions

$$(1.2) \qquad B^l[u] = \sum_{k+j=m} b^l_{kj}(z) \frac{\partial^{k+j} u}{\partial x^k \, \partial y^j} = f^l(z), \qquad l = 1, 2, \cdots, n \quad \text{on} \quad \kappa$$

where m is the same for all l, $n - 1 \leq m < 2n$ and where $b^l_{kj}(z)$ are analytic in a specific preassigned domain containing κ. We shall show that we get an explicit expression for the reflection across the analytic arc, provided an inequality depending on the b^l_{kj} is satisfied. Moreover, the domain into which we can reflect the solution can be expressed simply and explicitly in terms of (1) the arc, (2) the original region on which u is defined, and (3) the a_{ij} . Thus we have explicit reflection in the large. It is the use of complex variable methods that permits the simple determination of the region of reflection.

The equation (1.1) is a special case of the general class of elliptic equations that Garabedian considered in [2], however, his results are local and are not explicit.

In [4] the author has shown that is possible to reflect solutions of systems of the form

$$\Delta u + Au_x + Bu_y + Cu = 0,$$

$u = 1 \times n$ vector A, B, C constant pairwise commutative matrices, across an analytic arc on which the solutions satisfy analytic boundary conditions. A special example is the constant coefficient metaharmonic equation

$$\Delta^n u + a_{n-1} \Delta^{n-1} u + \cdots + a_1 u = 0.$$

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