WALLMAN AND Z-COMPACTIFICATIONS

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In [2], Frink generalized the compactification procedure of Wallman [7] to obtain Hausdorff compactifications of Tychonoff spaces. Frink uses a *normal base* of closed sets instead of the family of all closed sets as employed by Wallman. In [2], Frink asks whether all Hausdorff compactifications are obtainable by using normal bases. In his specific examples, the normal bases can be chosen as subrings of zero-sets. Frink has also asked whether this can always be done.

Recent papers by Njåstad [5], Hager [4], and one of the present authors [6] have shown that many compactifications are of the Wallman type as defined by Frink. We shall call these *Wallman compactifications*. The Wallman compactifications which arise from rings of zero-sets will be called *Z-compactifications*. We shall consider only Hausdorff compactifications.

§1 of this paper contains the necessary known results and uses the notation of [7].

In [7], a criterion was given for two families of closed sets to give the same Wallman compactification. In §2 we shall give a criterion for one Wallman compactification to be larger than another in the usual ordering on Hausdorff compactifications. We also give a necessary and sufficient condition for a given compactification to be Wallman. As applications, two new classes of compactifications are shown to be Wallman. In [2], it is shown that every one-point compactification is Wallman. It follows from the results of Njåstad [5], that every finite point compactification is Wallman. We shall show that every countable point compactification is Wallman.

§3 is concerned with whether a Wallman compactification is a Z-compactification. Our principal result is that a Wallman compactification which contains at most a countable number of multiple points (see [5]) is a Z-compactification. This includes the bounding system compactifications of Gould [3] and all finite and countable compactifications. We do not know whether all Freudenthal compactifications [1] are Z; however we show that they are in the special case of locally compact spaces.

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1. Preliminaries. Proofs of the results included here will be found in [7]. If a family $\mathfrak{F}$ of closed sets in X provides a Wallman compactification $\hat{X}$, we will write $\hat{X} = w(X, \mathfrak{F})$. The properties required of $\mathfrak{F}$ so as to produce a Wallman

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