A CLASS OF FORMULAS FOR THE RAYLEIGH FUNCTION

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1. Introduction. The Rayleigh function $\sigma_n(\nu)$ has been defined in a previous paper [1] in the following manner. Let $J_\nu(z)$ be the Bessel function of the first kind, and let $j_{\nu,m}$, $m = 1, 2, 3, \cdots$, be the zeros of $z^{-\nu}J_\nu(z)$ such that $|\mathrm{Re}(j_{\nu,m})| \leq |\mathrm{Re}(j_{\nu,m+1})|$. Then

$$\sigma_n(\nu) = \sum_{m=1}^{\infty} (j_{\nu,m})^{-2n}, \qquad n = 1, 2, 3, \cdots . \tag{1}$$

We remark that we have adopted here the symbol $\sigma_n(\nu)$ to represent the n-th Rayleigh function, whereas in [1] and [2] the same function was indicated by $\sigma_{2n}(\nu)$. The same holds for the n-th Rayleigh polynomial $\phi_n(\nu)$.

The function $\sigma_n(\nu)$ is a rational function of ν with rational coefficients [1], and that all of its real zeros [2; 915] lie in the interval $(-\,n, \,-\,2)$. A generating function for $\sigma_n(\nu)$ is [1; 6]

$$w = \tfrac{1}{2}J_{\nu+1}(z)/J_\nu(z) = \sum_{n=1}^{\infty} \sigma_n(\nu)z^{2n-1}. \tag{2}$$

The Rayleigh function $\sigma_n(\nu)$ is interesting in many ways; however, only a few formulas involving this function are so far known [1; 14, 20, 22, 26]. In the present paper we shall derive a differential equation which is satisfied by w; then from this differential equation we shall obtain a number of recurrence formulas for the function $\sigma_n(\nu)$. Setting $\nu = \pm\frac{1}{2}$, the corresponding recurrence formulas for the Bernoulli and Genocchi numbers may then be obtained [1; 4, 5].

2. Differential equation. Consider (2), which may be written as

$$2w = J_{\nu+1}(z)/J_\nu(z).$$

$$2w = \frac{\nu J_\nu(z) - zJ_\nu'(z)}{zJ_\nu(z)}, \qquad [3;\,45].$$

Differentiate with respect to z, then

$$2w' = \frac{zJ_\nu(z)\{(\nu - 1)J_\nu'(z) - zJ_\nu''(z)\} - \{zJ_\nu'(z) + J_\nu(z)\}\{\nu J_\nu(z) - zJ_\nu'(z)\}}{z^2J_\nu^2(z)}.$$

In the above substitute

$$z^2J_\nu''(z) = -zJ_\nu'(z) - (z^2 - \nu^2)J_\nu(z),$$

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