BACKWARD CONTINUOUS DEPENDENCE FOR MIXED PARABOLIC PROBLEMS

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1. Introduction. In the last few years many papers have appeared which have been devoted to uniqueness of solutions to initial value problems in various contexts in which corresponding existence theorems are, in general, not valid (for a representative sample of the literature see [1], [2], [4], [5], [6].). In particular, Lions and Malgrange [6] established such a uniqueness theorem for weak solutions of an abstract Cauchy problem in Hilbert space corresponding to the (non-well-posed) backward parabolic initial-boundary value (mixed) problem. The purpose of this paper is to establish a continuous dependence theorem under essentially the same assumptions on the equation as those in [6], but with more regular solutions. As a corollary we obtain a new proof of the uniqueness result of Lions and Malgrange in this situation. The proof is a development of that employed by Glagoleva [3] who established there the result for self-adjoint second order parabolic equations with coefficients independent of time (although the generalization to arbitrary second order parabolic equations was announced) and Dirichlet boundary conditions.

2. Assumption and results. Let V and H be two complex Hilbert spaces with $V \subseteq H$. The inner product and norm on V (respectively on H) will be denoted by $((\cdot, \cdot))$ and $||\cdot||$ (respectively $(\cdot, \cdot)$ and $|\cdot|$). V is supposed to be dense in H, and the embedding continuous (hence $|u| \leq h||u||$ for all $u \in V$).

For each $t \in [0, -T]$ let $a(t; u, v)$ be a sesquilinear form (linear in u, conjugate-linear in v) defined and continuous on $V \times V$. We suppose it is possible to write

$$a(t; u, v) = a_0(t; u, v) + a_1(t; u, v) \tag{1}$$

where the $a_i(t; u, v)$ $(i = 0, 1)$ are continuous sesquilinear forms for each $t \in [0, -T]$ which also satisfy the following further conditions:

(i) For all $u, v \in V$ and $t \in [0, -T]$, $a_0(t; u, v) = \overline{a_0(t; v, u)}$.

(ii) For all $u \in V$, and $t \in [0, -T]$,

$$a_0(t; u, u) \leq C_1 \, ||u||^2. \tag{2}$$

(iii) There are constants λ and $\alpha > 0$ such that for all $u \in V$ and $t \in [0, -T]$,

$$a_0(t; u, u) + \lambda \, |u|^2 \geq \alpha \, ||u||^2. \tag{3}$$

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