AN INVERSION THEOREM FOR THE LAPLACE TRANSFORM

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1. Introduction and main results. The Laplace–Stieltjes transform $f(s)$ of a function $\alpha(t)$ of bounded variation in each finite interval $[0, R]$, $R > 0$, is defined by

$$f(s) = \int_0^\infty e^{-st}\, d\alpha(t) = \lim_{R\uparrow\infty} \int_0^R e^{-st}\, d\alpha(t).$$

The Laplace transform $f(s)$ of a function $\varphi(t) \in L_1(0, R)$ for each $R > 0$, is defined by

$$f(s) = \int_0^\infty e^{-st}\varphi(t)\, dt = \lim_{R\uparrow\infty} \int_0^R e^{-st}\varphi(t)\, dt.$$

We suppose that the abscissa of convergence σ_c satisfies $\sigma_c > -\infty$. In a recent paper Bilodeau [1, Theorem 3.2], obtained an inversion formula for the Laplace transform $f(s)$ expressing $\varphi(x)$ for $x > 0$ by means of the sequence $\{f^{(n)}(1)\}$ $(n \geq 0)$ if $\sigma_c < 1$ and $\varphi(t)$ is of bounded variation in a neighborhood of x and $t^{-3/4}\varphi(t) \in L_1(0, \epsilon)$ for some $\epsilon > 0$. In this paper we obtain an inversion formula for the Laplace–Stieltjes transform expression $\alpha(x)$ and $\alpha^{(r)}(x)$ (if it exists for some $x > 0$) by means of the sequence $\{f^{(n)}(x_0)\}$ $(n \geq 0)$ where x_0 is any point satisfying $x_0 > \sigma_c$.

The formal idea used in this paper is the following. Suppose $\alpha(t)$ is absolutely continuous, $\varphi(t) \equiv \alpha'(t)$ and that $h_n(z, a)$ is an entire function in z for each $n \geq 0$ and each a, $0 \leq a < \infty$. For $x_0 > \sigma_c$ define

$$e^{x_0 z} h_n(z, a) \equiv \sum_{m=0}^\infty a_{nm}(a) z^m.$$

Then we have formally

$$\sum_{m=0}^\infty (-1)^m a_{nm}(a) f^{(m)}(x_0) = \sum_{m=0}^\infty a_{nm}(a) \int_0^\infty t^m e^{-x_0 t}\varphi(t)\, dt$$
$$= \int_0^\infty h_n(t, a)\varphi(t)\, dt.$$

If the sequence $\{h_n(t, a)\}$ $(n \geq 0)$ is a suitable kernel of a singular integral [6], then

$$\lim_{n\to\infty} \int_0^\infty h_n(t, a)\varphi(t)\, dt = \varphi(a);$$

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