ON A SINGULAR HYPERBOLIC OPERATOR

BY L. E. PAYNE AND D. SATHER

1. Introduction. The first maximum principles for linear second order hyperbolic operators in two independent variables were obtained for problems in which conditions are imposed on the solution along characteristic curves [1]; [5]. Various maximum properties for linear second order hyperbolic operators have also been established for Cauchy's problem in which conditions on the solutions are imposed on non-characteristic curves [6]; [9]; [11]; [12]; [13]; [14]; [15]; [16]; [17].

Recently, one of the authors [10] formulated maximum and monotonicity properties of some initial-boundary value problems for hyperbolic operators of the form

$$Hu = u_{yy} - h^2(x, y)u_{xx} + a(x, y)u_x + b(x, y)u_y + c(x, y)u. \tag{1.1}$$

For example, under certain restrictions on the coefficients of the operator H, if $h > 0$ in the closure $\bar{T}$ of a domain T bounded by $x = 0$, $y = 0$ and a characteristic curve of the operator H with everywhere negative slope and if $u \leq 0$ on $\{x = 0\} \cap \bar{T}$, $u = 0$ and $u_y \leq 0$ on $\{y = 0\} \cap \bar{T}$, and $Hu \leq 0$ in T then

$$u \leq 0 \quad \text{in} \quad T. \tag{1.2}$$

The requirement that h be positive in $\bar{T}$ is essential in the proof of (1.2) given in [10]. Thus the maximum property (1.2) would not necessarily hold in the important special case of (1.1) when $a = b = c = 0$ and $h(y) = y^\beta$ $(\beta > 0)$.

In this paper we obtain various maximum, monotonicity and convexity properties, as well as pointwise bounds, for the solution of some Cauchy and initial-boundary value problems for classes of linear second order hyperbolic operators in two independent variables. For the sake of simplicity we consider only the operator defined by the Chaplygin equation

$$u_{yy} - h^2(y)u_{xx} = 0, \tag{1.3}$$

namely,

$$Lu = u_{yy} - h^2(y)u_{xx}, \tag{1.4}$$

where

$$h(0) = 0 \tag{1.5}$$

and

$$h'(y) > 0 \qquad y > 0. \tag{1.6}$$

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