RETRACTION ONTO A DENDRITE

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1. Introduction. Borsuk has shown that a dendrite (acyclic curve) is a retract of any containing metric space [1]. In this paper we show that a dendrite is a semi-monotone retract [3] of any containing finite-coherent Peano space (compact, connected, locally connected metric space). A result of Wallace [4] is that an arc is a quasi-monotone retract of any containing Peano space. We show by an example that a dendrite is not always a quasi-monotone retract. The reader will find most unfamiliar terms in [5].

2. Main result. Let $r(X)$ denote the degree of multicoherence [5; 83] of the continuum (compact, connected metric space) X, and let $\mathcal{B}_X(A)$, $\mathcal{K}_X(A)$, and $X - A$ denote, respectively, the boundary, closure, and complement of A with respect to X.

Definition 1. Let $f(X) = Y$ be continuous:

a) f is said to be monotone iff $f^{-1}(y)$ is connected for all $y \in Y$ [5; 70].
b) f is said to be quasi-monotone iff for any continuum $C \subset Y$ with a nonvacuous interior, $f^{-1}(C)$ has a finite number of components and each of these maps onto C under f ([4], [5; 151]).
c) f is said to be semi-monotone iff $f^{-1}(C)$ has a finite number of components for every subcontinuum $C \subset Y$ [3; 755].

Theorem 1. *Let $r(X)$ be finite, where X is a Peano space and let D be a dendrite in X. Then there is a semi-monotone retraction $f(X) = D$.*

The proof will rely on middle-space topology where the middle spaces are regular curves. A continuum is a regular curve if there exist arbitrarily small neighborhoods of each point with finite point sets as boundaries [5; 96].

3. Finite-coherent regular curves.

Definition 2. A non-degenerate subcontinuum A of a continuum X is called

a) an A-set if $A = X$ or every component of $X - A$ has a single point as boundary [5; 67].
b) a B-set if $A = X$ or every component of $X - A$ has a finite point set as boundary [2; 122].

Theorem 2. *Let X be a continuum with a finite degree of multicoherence. Then X is a regular curve iff every non-degenerate subcontinuum is a B-set.*

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