HOMOMORPHISMS OF TOPOLOGICAL TRANSFORMATION GROUPS INTO FUNCTION SPACES

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1. Introduction. Let T be a locally compact abelian topological group and let Y be a topological space. The topological space $C(T, Y)$ of all continuous functions from T to Y in the compact-open topology becomes the phase space of a topological transformation group $\mathbf{F}(Y) = (C(T, Y), T, \sigma)$ when one defines $\sigma : C(T, Y) \times T \to C(T, Y)$ by $\sigma(f, t)(s) = f(ts)$. Using the local compactness of T, one shows that σ is continuous.

In this paper, we will study homomorphisms (that is, continuous equivariant maps) of topological transformation groups with phase group T and Hausdorff phase space into $\mathbf{F}(Y)$. Let $\mathbf{X} = (X, T, \pi)$ be a topological transformation group and Hom $(\mathbf{X}, \mathbf{F}(Y))$ be the collection of all (continuous) homomorphisms of $\mathbf{X}$ into $\mathbf{F}(Y)$ in the compact-open topology (a subspace of $C(X, C(T, Y))$). In §2, it is proved that the correspondence from $C(X, C(T, Y))$ into $C(X, Y)$ obtained by evaluating each function at the identity element of T yields a homeomorphism between Hom $(\mathbf{X}, \mathbf{F}(Y))$ and $C(X, Y)$. Further, this correspondence defines a covariant functor as X is allowed to vary over a category of topological transformation groups.

In §3, we show that the problem of extending homomorphisms into $\mathbf{F}(Y)$ can be reduced to the problem of extending a continuous map into Y. Examples are given for specific choices of the category of topological transformation groups and for Y.

The concepts of absolute retract and absolute neighborhood retract have been studied for various categories of topological spaces. Borsuk originally defined these concepts for compact metric spaces [1] and [2]. Later, Lefschetz [10], Dugundji [4], and Hu [6] extended these ideas to categories of separable metric, metric, and Tychonoff spaces. In [5], Hanner presents a comprehensive summary of the theory of retraction and extension and phrases the definitions so that the class of spaces involved is arbitrary. The recent book by Hu [8], contains this background material. In an analogous way, we define in §4 absolute retract and absolute neighborhood retract for a category of topological transformation groups. It is then shown that $\mathbf{F}(Y)$ has these properties under certain assumptions on Y and the category.

2. Definitions and notations. Category isomorphism theorem. Let $\mathcal{E}$ denote a category of topological transformation groups each having the same phase group T (which we assume to be locally compact and abelian) and each having

Received September 1, 1965.