THE SET OF ASYMPTOTIC VALUES OF A BOUNDED HOLOMORPHIC FUNCTION

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1. It is known [3], [4] that the set of asymptotic values, or briefly, *the asymptotic set*, of a function meromorphic in the unit disc is characterized as being an analytic set in the extended complex plane. Subsequently Heins [2] characterized the asymptotic set of an entire function as being an analytic set containing the point at infinity. A natural question to pose is that of characterizing the asymptotic set of a function holomorphic in the unit disc. In [3] Kierst constructed a function holomorphic in the unit disc whose set of *finite* asymptotic values is a prescribed analytic set in the finite plane; however, Kierst's function necessarily has infinity as an asymptotic value. Now, on the one hand, there exist functions holomorphic in the unit disc whose asymptotic sets do not contain infinity, while, on the other hand, there exist analytic sets which cannot be the asymptotic set of any function holomorphic in the unit disc, for instance the analytic set $\{0 \leq x \leq 1\}$. In view of these contrasting facts, our object in this note will be to characterize the asymptotic set of a holomorphic function mapping the unit disc onto itself. In a later note we will extend this characterization to unrestricted functions holomorphic in the unit disc.

Our present program calls first for establishing some necessary properties which asymptotic sets of functions holomorphic in the unit disc must possess. It is not known at this time whether these conditions are also sufficient for a characterization. Moreover, it does not seem likely that a characterization can be obtained by adding to these necessary conditions a topological property such as some type of connectivity or compactness. For example, it is possible to construct a bounded domain whose set of accessible boundary points is totally disconnected. Thus, using the universal covering surface of this domain, we obtain a bounded function holomorphic in the unit disc whose asymptotic set is totally disconnected. We conclude with a simple characterization of the asymptotic set of a bounded function holomorphic in the unit disc with radial limits of modulus one a.e.

2. We will use the following notations. If S is a set, then S^- and ∂S denote its closure and boundary. For complex-valued functions f defined in $\mathfrak{U} = \{|z| < 1\}$, the asymptotic set will be denoted $\mathfrak{A}(f)$.

The following theorem provides some necessary conditions on $\mathfrak{A}(f)$ for f holomorphic in $\mathfrak{U}$.

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