AN EMBEDDING THEOREM FOR DISCRETE FLOWS ON A CLOSED 2-CELL

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1. Introduction. The purpose of this paper is to prove that (1) the *restricted embedding problem* [3] for the closed 2-cell has a solution in case the discrete flow is almost periodic and orientation-preserving, and (2) the solution is unique if the discrete flow is strictly almost periodic. The first result strengthens a theorem in [3]. The embedding flow is obtained by applying a characterization of almost periodic homeomorphisms on the 2-cell obtained by the author in [2].

2. Embedding of discrete flows on a closed 2-cell. A topological transformation group $(X, \mathcal{G}, \pi)$ is said to be a *continuous flow* or a *discrete flow* on X according as the phase group $\mathcal{G}$ is the reals $\mathcal{R}$ or the integers $\mathcal{I}$ with the usual topology [5].

The *restricted embedding problem* [3] for discrete flows in continuous flows is that of finding a continuous flow $(X, \mathcal{R}, \pi)$ corresponding to a given space X and discrete flow $(X, \mathcal{I}, \sigma)$ such that $\pi(x, 1) = \sigma(x, 1)$ for all $x \in X$. The continuous flow $(X, \mathcal{R}, \pi)$ is then called an *embedding flow* for the discrete flow $(X, \mathcal{I}, \sigma)$ and the discrete flow $(X, \mathcal{I}, \sigma)$ is said to be *embedded in* the continuous flow $(X, \mathcal{R}, \pi)$.

The restricted embedding problem has been solved by Fort [4] for X an interval and by Foland and Utz [3] for X a simple closed curve. The problem is only partially solved for X a closed 2-cell. An embedding flow is obtained in [3] for a *periodic* discrete flow on the closed 2-cell.

The orbit of a point under a continuous flow on a 2-cell is either a point, an open arc, or a simple closed curve [1]. Thus a necessary condition for the embedding of a discrete flow $(X, \mathcal{I}, \sigma)$ is that each orbit under $\mathcal{I}$ be on an invariant subset of X of this type. This is trivially true in the case of the interval or the simple closed curve, but appears to be difficult to determine in the higher dimensional case.

An obvious necessary restriction on the discrete flow is that it be orientation preserving.

THEOREM 1. *Let $(X, \mathcal{I}, \sigma)$ be an almost periodic and orientation preserving discrete flow on a closed 2-cell X. Then there is a continuous flow $(X, \mathcal{R}, \pi)$ such that $\pi(x, 1) = \sigma(x, 1)$ for all $x \in X$.*

Proof. In [2] it is shown that any orientation-preserving almost periodic homeomorphism on X is topologically equivalent to a rotation of a disk about its center. Since $(X, \mathcal{I}, \sigma)$ is almost periodic and orientation-preserving, the map

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