THE PRODUCT OF AN UNUSUAL DECOMPOSITION SPACE WITH A LINE IS E^4

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I. Introduction. In (4) Bing describes a certain u.s.c. (upper semicontinuous) decomposition of E^3 which has only countably many nondegenerate elements, each of which is a point-like, indecomposable continuum; he proves that Y, the hyperspace of this u.s.c. decomposition of E^3, is not homeomorphic with E^3. Bing then asks whether or not $Y \times E^1 = E^4$. Here we answer this question affirmatively, using a method suggested by J. J. Andrews. As usual we define a sequence of homeomorphisms which gradually shrink certain subsets of E^4. However, we define these homeomorphisms by equations; we do not use cells as in [1] and [3]. It is interesting to note that the homeomorphisms are not uniformly continuous. The reader may consult [2] for further background.

II. Preliminaries. For basic definitions see [5] or [6]. If X is a metric space, d its distance function, A a subset of X and $\epsilon > 0$, then by $S(A, \epsilon)$ we mean $\{x \in X : d(x, A) < \epsilon\}$. For a subset B of a topological space X by ∂B we mean the topological boundary of B; by cl (B) we mean the closure of B; and by int (B) we mean the interior of B. If $Z = \{A_i\}$ where each $A_i \subset X$, then $Z^* = \cup A_i$. By id. we mean the identity function. We take $E^2 = E^2 \times 0 \subset E^2 \times E^1 = E^3$, etc.

III. A useful homeomorphism on E^3. Let points in E^2 be given in polar coordinates (r, θ). Let A be the plane annular region given by $\{(r, \theta) : \frac{1}{4} \leq r \leq \frac{3}{4}\}$ and let S be the circle of radius $\frac{1}{2}$ with center at the origin. For each real number ϕ define $T_\phi : E^2 \to E^2$ as follows:

$$T_\phi(r, \theta) = \begin{cases} (r, \theta) & \text{if } r \leq \frac{1}{8} \text{ or } r \geq \frac{7}{8} \\ (r, \theta - 8\phi(r - \frac{1}{8})) & \text{if } \frac{1}{8} \leq r \leq \frac{1}{4} \\ (r, \theta - \phi) & \text{if } \frac{1}{4} \leq r \leq \frac{3}{4} \\ (r, \theta - 8\phi(\frac{7}{8} - r)) & \text{if } \frac{3}{4} \leq r \leq \frac{7}{8}. \end{cases}$$

It is routine to check that T_ϕ is continuous, and is actually a homeomorphism of E^2 onto E^2.

Now let $E^2 = E^2 \times 0 \subset E^2 \times E^1$. Let $B = \{x \in E^3 : d(x, S) \leq \frac{1}{4}\}$, which is a solid torus containing A. Let $T = \{x \in E^3 : d(x, S) \leq \frac{3}{8}\}$, so that T is a

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