GENERALIZED TEMPERATURE FUNCTIONS

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1. Introduction. Our goal is a characterization for generalized temperature functions which are represented by a Poisson-Hankel-Stieltjes integral transform $\int_0^\infty G(x, y; t)\, d\alpha(y)$. A set of necessary and sufficient conditions for such a representation was given in [2], but these results parallel those in [6] for the ordinary temperature function. Further, we derive an inversion formula for the Poisson-Hankel transform, establish the growth behavior of a generalized temperature function, determine that a function has the Huygens property if it is represented by a convergent Poisson-Hankel-Stieltjes integral, and obtain a monotonic property for certain integrals involving functions with the Huygens property.

2. Definitions and preliminary results. We need the following basic definitions.

Definition 2.1. The generalized heat equation is

$$\Delta_x u(x, t) = \frac{\partial}{\partial t} u(x, t),$$

where $\Delta_x f(x) = f''(x) + (2\nu/x)f'(x)$, ν a fixed positive number. Note that if $\nu = 0$, this reduces to the ordinary heat equation.

Definition 2.2. The fundamental solution of the generalized heat equation is the function

$$G(x, y; t) = (1/2t)^{\nu+\frac{1}{2}} \exp\left[-(x^2 + y^2)/4t\right] \mathcal{J}(xy/2t),$$

where $\mathcal{J}(z) = c_\nu z^{1/2-\nu} I_{\nu-1/2}(z)$, $I_\alpha(z)$ being the Bessel function of imaginary argument of order α, and $c_\nu = 2^{\nu-1/2}\Gamma(\nu + \frac{1}{2})$. We write $G(x; t)$ for $G(x, 0; t)$.

Definition 2.3. A generalized temperature function is a function of class C^2 which satisfies the generalized heat equation. We denote the class of such functions by H.

Definition 2.4. A generalized temperature function $u(x, t)$ such that

$$u(x, t) = \int_0^\infty G(x, y; t - t')u(y, t')\, d\mu(y), \qquad d\mu(x) = c_\nu^{-1} x^{2\nu}\, dx,$$

the integral converging for every t, t', $a < t' < t < b$, is said to have the Huygens property for $a < t < b$. We denote by H^* the class of such functions.

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