PACKINGS WITH LACUNAE

By Norman Oler

The purpose of this note is to show how the concept of a packing with respect to a plane convex disk as we have defined it [1; 20] may be generalized to the packing with respect to such a disk of disconnected, multiply connected domains. Moreover, the slackness function introduced by Zassenhaus [3; 432] has a corresponding extension which we shall show is again non-negative.

We recall that a packing with respect to a plane convex disk K is a pair (π, E) in which

1) E is a finite set;
2) π is a Jordan polygon whose vertices belong to E and which contains the remaining points of E in its interior, π^*;
3) m being the Minkowski distance determined by K, the m-length of any segment with end points in E which lies in $\overline{\pi^*}$ is not less than 1.

The extension of this definition which we wish to make is to remove the condition that π be a Jordan polygon but that it satisfy the following.

π has components $A(1)$, $B(1, 1)$, $\cdots$, $B(1, k_1)$; $\cdots$; $A(r)$, $B(r, 1)$, $\cdots$, $B(r, k_r)$ each of which is a Jordan polygon and which satisfy:

$$\overline{A^*(i)} \cap \overline{A^*(j)} = \phi \qquad 1 \leq i < j \leq r;$$

$$B^*(i, s) \subset A^*(i) \qquad 1 \leq s \leq k_i\,, \qquad 1 \leq i \leq r;$$

$$\overline{B^*(i, s)} \cap \overline{B^*(i, t)} = \phi \qquad 1 \leq s < t \leq k_i\,, \qquad 1 \leq i \leq r.$$

In the above conditions on (π, E), π^* is to be taken as

$$\bigcup_{i=1}^{r}\left[A^*(i) - \bigcup_{j=1}^{k_i}\overline{B^*(i, j)}\right].$$

We allow that for any i, $\{B(i, j)\}$ may be empty in which case $k_i = 0$ and $\bigcup_{j=1}^{k_i}\overline{B^*(i, j)} = \phi$. The slackness function S is now defined by

$$S(\pi, E) = \frac{\alpha(\pi^*)}{\Delta} + \frac{M(\pi)}{2} + r - \sum_{i=1}^{r} k_i - N$$

in which Δ is the critical determinant of K, $\alpha(\pi^*)$ is the area of π^*, $M(\pi)$ is the m-length of π and N is the number of points in E.

Theorem. *For any packing, (π, E), $S(\pi, E)$ is non-negative.*

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