SOME CONTINUED FRACTIONS OF THE ROGERS-RAMANUJAN TYPE

By Basil Gordon

1. **Introduction.** The continued fraction of Rogers and Ramanujan is

$$1 + \frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+\cdots}. \tag{1}$$

Using the Rogers–Ramanujan identities it can be shown that for $|x| < 1$, this fraction is equal to the product

$$\prod_{n=1}^{\infty} (1 - x^{5n-3})(1 - x^{5n-2})/(1 - x^{5n-4})(1 - x^{5n-1})$$

(cf [5; 290–295]). This result, together with the theory of singular moduli, enabled Ramanujan to evaluate (1) when $x = \exp(-\pi\sqrt{r})$, r any positive rational number.

In this paper we consider a number of continued fractions analogous to (1), of which the following is typical:

$$1 + x + \frac{x^2}{1 + x^3 +}\,\frac{x^4}{1 + x^5 +}\,\frac{x^6}{1 + \cdots}. \tag{2}$$

For these fractions we obtain the results corresponding to those stated above. In each case the numerator and denominator are evaluated by using identities of the Rogers–Ramanujan type, several of which are due to Slater [7], [8]. Some of these identities have combinatorial interpretations similar to those given by MacMahon and Schur [6] for the Rogers–Ramanujan identities themselves. The fractions can then be evaluated whenever $x = \pm \exp(-\pi\sqrt{r})$ by a method similar to that used by Ramanujan (for a description of this see [9]). The following theorems concerning (2) will serve as an illustration.

Theorem 1. *If $|x| < 1$, then (2) converges to the value*

$$\prod_{n=1}^{\infty} (1 - x^{8n-5})(1 - x^{8n-3})/(1 - x^{8n-7})(1 - x^{8n-1}).$$

Theorem 2. *The number of partitions of any positive integer n into parts $\equiv 1, 4$, or 7 (mod 8) is equal to the number of partitions of the form $n = n_1 + n_2 + \cdots + n_k$, where $n_i \geq n_{i+1} + 2$, and $n_i \geq n_{i+1} + 3$ if n_i is even $(1 \leq i \leq k - 1)$.*

Theorem 3. *The number of partitions of n into parts $\equiv 3, 4$, or 5 (mod 8) is equal to the number of partitions $n = n_1 + \cdots + n_k$ satisfying $n_k \geq 3$ in addition to the inequalities of Theorem 2.*

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