A CLASS OF MONOTONE RICCATI MATRIX DIFFERENTIAL OPERATORS

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1. Introduction. In the study of conditions for the "scattering matrix" of a generalized Mycielski-Paszkowski diffusion problem to be dissipative in the sense of Redheffer [8], the author [9] has established sufficient conditions for a solution $W(t)$ of the Riccati matrix differential equation with real coefficients

$$(1.1) \qquad W' = A(t) + D(t)W + WB(t) + WC(t)W, \qquad s \leq t \leq s_1 ,$$

to have non-negative elements throughout $[s, s_1]$ whenever $W(s)$ has non-negative elements. The prime consideration of the present paper is the study of some allied problems for matrix differential equations, including, in particular, necessary and sufficient conditions for (1.1) to possess the above-stated property, and conditions for the extensibility of such solutions on the entire interval $[s, \infty)$. Also, for the special equation (1.1) with A, B, D identically zero there are established results which extend classical results of Perron and Frobenius on the dominant proper value, and associated proper vector, of an irreducible, non-negative constant matrix.

For a general $m \times n$ matrix $M = [M_{\alpha\beta}]$ the symbol $M \cdot\geq\cdot 0$, $\{M\cdot>\cdot 0\}$, will signify that the elements of M are real and $M_{\alpha\beta} \geq 0$, $\{M_{\alpha\beta} > 0\}$, for $\alpha = 1, \cdots, m; \beta = 1, \cdots, n$. If M is a real square matrix, the symbol $M*\geq*0$ will signify that $M_{\alpha\beta} \geq 0$ for $\alpha \neq \beta$; the symbols $M\cdot\leq\cdot 0$, $M\cdot<\cdot 0$, and $M*\leq*0$ will denote the respective conditions $(-M)\cdot\geq\cdot 0$, $(-M)\cdot>\cdot 0$, and $(-M)*\geq*0$. Also, $M\cdot\geq\cdot N$ will signify $M - N\cdot\geq\cdot 0$, with similar meanings for $M\cdot>\cdot N$, $M*\geq*N$, $N\cdot<\cdot M$, etc. The positive orthant $\{y \mid y_\alpha \geq 0, \alpha = 1, \cdots, n\}$ will be denoted by $\mathfrak{H}$, $\mathfrak{M}$ will signify the set of $n \times n$ real matrices $\{M \mid M = [M_{\alpha\beta}], M_{\alpha\beta} \text{ real}\}$, and $\mathfrak{M}^+ = \{M \mid M \in \mathfrak{M}, M\cdot\geq\cdot 0\}$.

For $y \in \mathfrak{H}$ we define $|y| = \sum_{\alpha=1}^n y_\alpha$; for $M \in \mathfrak{M}^+$ we set $\mu(M) = \sup |My|$ and $\nu(M) = \inf |My|$ on $\{y \mid y \in \mathfrak{H}, |y| = 1\}$; if $m^{(1)}, \cdots, m^{(n)}$ denote the column vectors of M, then clearly $\mu(M)$ is the maximum, and $\nu(M)$ the minimum, of the values $|m^{(1)}|, \cdots, |m^{(n)}|$. If the elements of $M(t)$ are continuous on an interval $\mathfrak{T}$, then the scalar functions $\mu(M(t))$ and $\nu(M(t))$ are continuous on this interval. Moreover, if M_1 and M_2 belong to $\mathfrak{M}^+$, then $\mu(M_1M_2) \leq \mu(M_1)\mu(M_2)$, $\mu(M_1 + M_2) \leq \mu(M_1) + \mu(M_2)$, $\nu(M_1M_2) \geq \nu(M_1)\nu(M_2)$, $\nu(M_1 + M_2) \geq \nu(M_1) + \nu(M_2)$, while $\mu(cM) = c\mu(M)$ and $\nu(cM) = c\nu(M)$ if $M \in \mathfrak{M}^+$ and $c \geq 0$.

The symbol E is used for the $n \times n$ identity matrix, while 0 is used indis-

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