WIEGMANN TYPE THEOREMS FOR *EPr* MATRICES

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1. **Introduction.** Let F be a field and let $\lambda : a \to \bar{a}$ be an involutory automorphism of F [and the identity mapping is considered to be an involutory automorphism of F]. For an $n \times n$ matrix $A = (a_{ij})$ with elements from F, let $A^* = (b_{ij})$ where $b_{ij} = \bar{a}_{ji}$. In the case where F is the complex field K, the automorphism is taken to be complex conjugation.

Let $\eta(A)$ denote the null space of A. An $n \times n$ matrix A of rank r is called *EPr* if $\eta(A) = \eta(A^*)$. *EPr* matrices were introduced by H. Schwerdtfeger [4] as a generalization of symmetric matrices and later studied by M. Pearl [2], [3]. In these papers attention has been focused on a single *EPr* matrix. It is the purpose of the present investigation to obtain some sufficient conditions that the product of an EPr_1 and an EPr_2 matrix be an *EPr* matrix.

2. **The complex case.** If A is an EPr_1 matrix and B is an EPr_2 matrix, then AB may not be an *EPr* matrix. For example, over C,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

are normal matrices of rank 1 and hence EP_1. But the product

$$C = AB = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

is not EP_1 since $C \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$, but $C^* \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The main purpose of this section is to prove Theorem 1:

If A is a complex EPr_1 matrix and B is a complex EPr_2 matrix such that $AB = BA$, then AB is an EPr matrix.

Let $V \subseteq K_n$ and let $V^\perp$ denote the orthogonal complement of V under the usual inner product. The following is proved by a direct computation.

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