UNIFORM CROSS NORMS AND TENSOR PRODUCTS OF BANACH ALGEBRAS

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1. Kronecker products. The purpose of the present paper is to provide some of the details of the results announced in [5]. We refer the reader to that paper on matters concerning motivation and background, as well as for some of the notations and definitions. We begin our discussion with the following lemma, which will be used in many places below, but which we state without proof.

Lemma 1.1. *Let E and F be Banach spaces and $T : E' \to F$ a bounded linear transformation which is continuous with respect to the weak* topology of E' and the weak topology of F. Suppose that M_1 and M_2 are respective subsets of E' and F', each of which is total with respect to the weak* topology. Finally, let us assume that for every $x' \in M_1$ and $y' \in M_2$,*

$$\langle Tx' , y' \rangle = 0. \tag{1.1}$$

Then $T = 0$.

Gelbaum [1] (see also Tomiyama [10]) introduced a multiplication on the tensor product $A \otimes B$ of any two algebras A and B over the same field, turning $A \otimes B$ into an algebra over that field. The resulting multiplication is the bilinear extension to the entire $A \otimes B$ of the product

$$(U_1 \otimes V_1)(U_2 \otimes V_2) = U_1U_2 \otimes V_1V_2 \tag{1.2}$$

of generators of $A \otimes B$, where $U_1 , U_2 \in A$ and $V_1 , V_2 \in B$. The tensor product $A \otimes B$ will always be considered in this paper as an algebra under the product defined by (1.2). Let now $\mathcal{B}(E)$ denote the Banach algebra of all bounded linear operators on a Banach space E. Let $U \in \mathcal{B}(E)$ and $V \in \mathcal{B}(F)$, where F is also a Banach space. Consider a tensor t of the algebraic tensor product $E \otimes F$ with a representation

$$\mathrm{t} = \sum_{i=1}^{n} x_i \otimes y_i . \tag{1.3}$$

We define $U \otimes V(\mathrm{t})$ by the relation

$$U \otimes V(\mathrm{t}) = \sum_{i=1}^{n} Ux_i \otimes Vy_i . \tag{1.4}$$

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