OPEN SIMPLICIAL MAPPINGS OF MANIFOLDS ON MANIFOLDS

By Erik Hemmingsen

Dedicated to A. D. Wallace on his sixtieth birthday.

Introduction. Let f be an open simplicial mapping sending a compact n-manifold M (possibly with boundary) onto an n-manifold N (possibly with boundary). Let B_f, the branch set of f, be the set of points at which f fails to be a local homeomorphism. (In the terminology of H. Hopf [2], $f(B_f)$ is the set of points of positive defect.) Let the number, (local Brouwer degree at x) $- 1$, be called the exceptionality of f at x. The purpose of this note is to describe some properties of B_f in the case that the exceptionality of B_f or some of its subsets is maximal either globally or locally, and to mention a number of questions to which the answer is not known. Manifolds are assumed regular.

The appearance of H. Hopf's paper [2] has caused the author to examine some of those aspects of [2; 280-283] which are not touched upon by the work of Church and the author [1, 615ff]; that is the immediate motivation for the completion of this paper. Suggestions along some of these lines were made to the author in 1947 and 1948 by A. N. Milgram.

1. Terminology and preliminary remarks. Use will be made of the work of Tucker [4], especially of his formula

$$\mathrm{X}(M) + \sum e_i \mathrm{X}(K_i) = d\,\mathrm{X}(N),$$

which is applicable to the situation in which: $f : M \to N$ is a simplicial mapping which collapses no simplices but which may induce folding, M and N are compact orientable manifolds, d is the degree of f, X is the euler characteristic, and K_i is the set of open simplices of M where the exceptionality is e_i. The additional restriction that f be open removes the possibility of folding. When $n = 2$ and f is open, the Tucker formula coincides with the Hurwitz-Riemann formula. It is clear that the set B_f consists of the points at which the exceptionality is not zero.

Reference will be made to a result of H. Hopf [2; 281] which is stated here for the convenience of the reader: Let M and N be orientable closed manifolds, let $f : M \to N$ be an open simplicial mapping, let Δ_1 be the set $\{y : f^{-1}(y)$ is a single point$\}$, let the degree of f be relatively prime to the order of the one dimensional torsion group of M^n, and let b_i denote the i-dimensional betti number. Then

$$b_{n-2}(\Delta_1) \leq 1 + b_1(M) - b_1(N) + b_2(N);$$

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