CONVERGENCE OF COMPLEX LAGRANGE INTERPOLATION POLYNOMIALS ON THE LOCUS OF THE INTERPOLATION POINTS

By J. H. Curtiss

1. Introduction. Let C be a simple closed curve in the complex z-plane, let f be a continuous function from C to the complex numbers, let $S_n = \{z_{n0}, z_{n1}, \cdots, z_{nn}\}$ denote for each positive integer n a set of $n + 1$ distinct points on C, and finally let $L_n(f; z)$ be the polynomial of degree at most n found by interpolation to f at the points S_n. The classical Lagrange formula for L_n is

$$L_n(f; z) = \sum_{k=0}^{n} f(z_{nk}) \frac{\omega(z)}{(z - z_{nk})\omega'(z_{nk})}, \tag{1.1}$$

where $\omega(z) = (z - z_{n0})(z - z_{n1}) \cdots (z - z_{nn})$. The expression $\omega(z)/(z - z_{nk})$ becomes formally indeterminate when $z = z_{nk}$. Here, and wherever in the sequel artificial singularities are present, it will be assumed that definitions are completed by continuity.

It is known [4] [6] that if the sequence $S_1, S_2, \cdots$ is suitably chosen and C is suitably smooth, then for all such functions f,

$$\operatorname*{Lim}_{n\to\infty} L_n(f; z) = \frac{1}{2\pi i} \int_C \frac{f(t)}{t - z}\, dt = f_0(z) \tag{1.2}$$

for z on the interior region D of C. The successful choice of S_n is essentially the following one: Let

$$z = \varphi(w) = d\left(w + d_0 + \frac{d_1}{w} + \frac{d_2}{w^2} + \cdots\right), \qquad d > 0, \tag{1.3}$$

be univalent and analytic on the set $\{w: 1 < |w| < \infty\}$ and continuous on $\{w: 1 \leq |w| < \infty\}$; let it map $\{w: |w| > 1\}$ conformally onto the complement of $D \cup C$ so that $\infty \leftrightarrow \infty$; and let $S_n = \{\varphi(\exp(2\pi ik/(n+1))), k = 0, 1, 2, \cdots, n\}$. This point set S_n will be called the $(n + 1)$-th set of Fejér points on C, in recognition of certain important results obtained by L. Fejér using these points of interpolation. (See especially [9].) In the classical case in which f is analytic on $C \cup D$, the set S_n need only be the image of an equidistributed set on $|w| = 1$ rather than of an equispaced set [9], but the author recently showed [5] that if f is merely continuous on $C \cup D$, analytic on D, then equispacing on $|w| = 1$ is essential for (1.2).

We shall deal in this paper exclusively with interpolation in Fejér points.

Received November 21, 1963. This work was supported by the U.S. Air Force through the Air Force Office of Scientific Research under Grants AF-AFOSR 62-189 and 358-63.