LOCALLY FLAT IMMERSIONS AND WHITNEY DUALITY

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1. Introduction. Let M denote a topological n-manifold and S a topological $(n + k)$-manifold. Then, if $f : M \to S$ is a locally flat *imbedding* and $(\mathfrak{J}, \mathfrak{J}_0)$ is the tangent fiber space of M, it was shown in [1] that it is possible to associate with this imbedding a normal fiber space $(\mathfrak{N}, \mathfrak{N}_0)$ such that $(\mathfrak{J}, \mathfrak{J}_0) \oplus (\mathfrak{N}, \mathfrak{N}_0)$, where $\oplus$ is an appropriately defined Whitney sum, is fiber homotopy equivalent to the fibered pair over M induced by f and the tangent fiber space of S. $(\mathfrak{J}, \mathfrak{J}_0)$ and $(\mathfrak{N}, \mathfrak{N}_0)$ are in this case very nice fibered pairs, more precisely generalized n-plane and k-plane bundles [1]. Thus, if $W(M)$ and $W(S)$ are the total Stiefel-Whitney class of M and S, respectively, and $\bar{W}_f$ is that unique class in $H^*(M; Z_2)$ such that $W(M) \cup \bar{W}_f = f^*(W(S))$ ([7], [2]), then $\bar{W}_f$ has a geometric interpretation as the total Stiefel–Whitney class of a generalized k-plane bundle, namely $(\mathfrak{N}, \mathfrak{N}_0)$. The use of $(\mathfrak{N}, \mathfrak{N}_0)$ allows one to employ many of the techniques used in the differential analogue and also provides geometric results [1].

Our objective here is to develop such a theory for locally flat immersions (= locally, locally flat imbeddings), where the situation is, unfortunately, more complicated. In this context, the concept of generalized n-plane bundle [1] must be weakened a little. In addition, in order to recognize the weak homotopy type of certain spaces, the *c–o* topology in certain path spaces must be weakened (i.e., made finer by enlarging the topology) also. (The latter can have certain advantages, e.g. if $f : X \to Y$ is a map, where Y has a topology weaker than one usually employed, then f is, in some sense, "nicer". We will see a specific application of this in (3.2.2.).) We will associate with every locally flat immersion $f : M \to S$ a normal fiber space (§3) which is a homology k-plane bundle (§2) and prove the Whitney Duality Theorem for such immersions (§5). In particular, it will follow that if $g : M \to S$ is any map and $\bar{W}_g$ is defined by $W(M) \cup \bar{W}_g = g^*(W(S))$, then, if $g \sim f$, where f is a locally flat immersion, $\bar{W}_g$ is realizable as the total Stiefel–Whitney class of a homology k-plane bundle, namely, the normal fiber space associated with f.

Singular homology will be employed throughout.

2. Preliminaries.

DEFINITION (2.1). An *n-manifold M* is a connected separable metric space which is locally homeomorphic to R^n (Euclidean n-space).

DEFINITION (2.2). Let $f : M \to S$ denote a map, where M is an n-manifold

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