CONTINUOUS FLOWS WITH CLOSED ORBITS

BY TA-SUN WU

1. Introduction. A continuous one-dimensional flow (hereinafter called simply a flow) in a topological space X is a continuous mapping from $R \times X$ onto X (R denotes the real line) which has the group property:

$$\varphi(\alpha, \varphi(\beta, x) = \varphi(\alpha + \beta, x) \quad \text{for all} \quad \alpha, \beta \in R, \qquad x \in X.$$

Each point has an orbit $\vartheta(x) = \{\varphi(\alpha, x) \mid \alpha \in R\}$. We are interested in the set of points whose orbits consist of single points, that is, those fixed under the flow:

$$\varphi(\alpha, x) = x, \quad \text{all} \quad \alpha \in R.$$

This set of fixed points is called invariant set of φ. Anatole Beck had proved that if a metric space admits a flow without fixed points (i.e. the invariant set is empty), then any closed subset of the space can be the invariant set of some flow [1]. So without further conditions on spaces or flows, the invariant sets can be very complicated and might not have a general structural property. Recently Beck gave a structural theorem on the invariant sets for flows acting in the plane with closed orbits [2]. We pursue his study to the case where the space X is a connected two-dimensional manifold (hereinafter called simply surface). We are going to characterize the invariant sets for flows with compact orbits. As to the flows with closed orbits, the result follows by a method similar to Beck's technique [2].

2. Lifting of the flows. We first study the relation between the flows on a space and its universal covering space. This is done by the lifting procedure.

(2.1) LEMMA. *Let φ be a flow on a space X which has a universal covering space E. Then there is a flow $\varphi^{\sim}$ on E such that*

$$\pi\varphi^{\sim}(\alpha, e) = \varphi(\alpha, \pi(e)), \qquad \alpha \in R, \qquad e \in E$$

π, the projection (covering map): $E \to X$.

Proof. It is clear that $(R \times E, \pi')$, $(R \times R \times E, \pi'')$ are the universal covering spaces of $R \times X$, $R \times R \times X$ respectively where $\pi'((\alpha, e)) = (\alpha, \pi(e))$, $\pi''((\alpha, \beta, e)) = (\alpha, \beta, \pi(e))$. Choose a point x_0 in X, let $e_0 \in E$ such that $\pi(e_0) = x_0$. Let us consider the following diagram:

Received June 2, 1963.