NONABSORPTION PROBABILITY FOR A GAUSSIAN PROCESS IN THE KARHUNEN REPRESENTATION

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1. Introduction. The problem is to find the probability that sample functions $X(t)$ of a given (real-valued) Gaussian stochastic process satisfy the inequalities

$$a(t) \leq X(t) \leq b(t), \qquad 0 \leq t \leq T,$$

where $a(t)$ and $b(t)$ are two given "barrier" functions. The data of the process are given via its covariance function $r(s, t) = EX(s)X(t)$; we assume the means $EX(t) \equiv 0$.

Solutions, of one sort or another, have been known only for Markov processes, and for one or two others. The principal cases considered have been with $a(t)$ and $b(t)$ constant functions. For the "one-sided barrier" case $a(t) \equiv -\infty$ and usually $b(t) \equiv 0$. For a review of known results and a bibliography, we refer the reader to D. Slepian [11].

In this paper we consider $X(t)$ as given by its Karhunen expansion (see (3) below). We estimate the error involved in truncating such an expansion; and we indicate an asymptotic expansion for the zero-crossing probability for a process $Z(t) + X(t)$, where $Z(t)$ is of diffusion type.

The Karhunen "change of coordinates" has been used to great advantage in other problems (cf. Grenander [6], Kac and Siegert [7]) and it is natural in this setting, too—especially in conjunction with a simultaneous expansion of $a(t)$ and $b(t)$. Using these coordinates, one has more of an analytical hold in approximating the "nonabsorption probability" $P[a(t) \leq X(t) \leq b(t), 0 \leq t \leq T]$ than one has when using the direct approach involving

$$\lim_M P[a(t_i) \leq X(t_i) \leq b(t_i), 0 \leq t_1 \leq \cdots \leq t_M \leq T].$$

In §§2 and 4, we shall consider $X(t)$ which have a finite (N-term) Karhunen expansion. It is then easy to see how the nonabsorption probability depends on three distinct sets of input data: (1) the eigenvalues of $r(s, t)$ (considered as integral kernel), (2) the corresponding eigenvectors, (3) the boundaries. We can interpret nonabsorption probabilities in terms of a simple diffusion in N-space. The main theorem of §2 presents the nonabsorption probability for the sum $Z(t) + X(t)$ of two processes in terms of a convolution transform applied to the corresponding probability for $Z(t)$ alone.

In §3 we give an estimate for the error involved in truncating a Karhunen

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