CUTTINGS OF FINITE SETS OF POINTS

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1. Introduction. In the main, the development of cyclic element theory, in which the cutpoint plays the central role, was carried out two or three decades ago. An exhaustive treatment and bibliography is given in [10], by a principal originator. Extensions in several directions have been made; recently there has been a flurry of activity by Cesari and Neugebauer in developing a fine-cyclic element theory, using cuttings by finite sets of points in place of single points, in connection with the former's work on the theory of surface area. (See [1], [2], [5], [6], and [7]). This paper is concerned at first with unifying some of the extensions of cyclic element theory, and then with an examination of fine-cyclic element theory in a more general framework.

2. General considerations. Let S be an arbitrary connected Hausdorff space, and $\mathcal{C} = \{C_\gamma\}$ be a collection of non-empty closed sets. A C-nodal decomposition, denoted (A, C, B) is a decomposition of S into closed connected sets A and B, $S = A \cup B$, such that $A \cap B = C \in \mathcal{C}$. By convention we require that neither A nor B equals C, and call A and B C-nodal sets. A C-chain is a non-empty intersection of C-nodal sets. The name "chain" for this type set is due to Wallace (See [9]). If X is a non-empty subset of S, $C(X)$ is defined to be the intersection of all C-nodal sets containing X. Some of the immediate or nearly immediate properties of the C-operator are:

(1) $X \subset C(X)$,
(2) If $X \subset Y$, then $C(X) \subset C(Y)$,
(3) $C(C(X)) = C(X)$,
(4) $C(\cap C(X_\beta)) = \cap C(X_\beta)$, if $\cap C(X_\beta) \neq \phi$,

and

(5) $C(\cap(X_\beta)) \subset \cap C(X_\beta)$, if $\cap X_\beta \neq \phi$.

It is convenient but, as will be seen, by no means necessary, for this paper to consider a slight extension of these ideas. Let $\mathcal{E} = \{E_\epsilon\}$ be another family of subsets of S, not necessarily closed. A CE-decomposition is a decomposition $(A; C, E; B)$ of S into sets A and B with $A \cap B = C \cup E$, so that a CE-decomposition is the same as a decomposition $(A, C \cup E, B)$, except in allowing E to be not closed. A CE-chain is then a non-empty intersection of CE-nodal sets, prime if incapable of further subdivision in this way, and $CE(X)$ is the intersection of all CE-nodal sets containing X. It is clear that properties

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