A NEW SERIES TRANSFORM WITH APPLICATIONS TO BESSEL, LEGENDRE, AND TCHEBYCHEFF POLYNOMIALS

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1. Introduction. In a series of preceding papers (see references [3] through [9]), the writer has developed a series transform motivated by the addition theorem for binomial coefficients. It was shown that if we write

$$F(n, a, b, f) = \sum_{k=0}^{n} (-1)^k \binom{n}{k}\binom{a+bk}{n} f(k) , \tag{1.1}$$

where $f(k)$ is independent of n and $f(0) = 1$, then

$$\binom{a+bn}{n} f(n) = \sum_{k=0}^{n} (-1)^k A_{n-k}(a + bk - k, b) F(k, a, b, f), \tag{1.2}$$

with

$$A_k(a, b) = \frac{a}{a+bk}\binom{a+bk}{k}. \tag{1.3}$$

Moreover

$$\sum_{k=0}^{\infty} \binom{a+bk}{k} z^k f(k) = x^a \sum_{n=0}^{\infty} (-1)^n F(n, a, b, f)\left(\frac{x-1}{x}\right)^n, \tag{1.4}$$

with $z = (x - 1)/x^b$.

These results were suggested by the generalized addition theorem (or what we have also called a generalized Vandermonde convolution)

$$\sum_{k=0}^{n} (p + qk) A_k(a, b) A_{n-k}(c, b) = \frac{p(a+c) + qan}{a+c} A_n(a + c, b). \tag{1.5}$$

The object of the present paper is to modify the transform (1.1) slightly so as to have a new transform widely applicable to various special functions, and in particular we shall find novel results for the Bessel polynomials of Krall and Frink [10], and for both Legendre and Tchebycheff polynomials. The emphasis in the preceding papers was on the study of identities for binomial coefficients, however it is not at all surprising that the work may be extended to various orthogonal polynomials and special functions since these involve the binomial coefficients in many of the identities they satisfy. In the present paper, for instance, we find a binomial coefficient summation which is readily transformed into a convolution of Bessel polynomials. The transform we

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