A CONVERSE FORM OF DAÏOVITCH'S THEOREM

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Daïovitch's theorem [3] is a special form of the Hadamard product theorem [4]. This paper is devoted to proving a converse form of Daïovitch's theorem by considering analytic functions which have positive real coefficients in their Maclaurin-series representation. The principal theorem in this presentation is identified as Theorem 3.

We shall be concerned with classes of functions defined by G. H. Hardy [5] and their corresponding Lebesgue classes. The functions $f(z)$ considered in the following definitions and lemmas shall be singled-valued and analytic within an open circle in the complex plane which has unit radius and center at the origin. Basic definitions and lemmas, which we shall need in the proof of our main result, follow.

The statement that $f(z)$ belongs to class B means that $f(z)$ is bounded in the unit circle, $|z| < 1$.

For $p > 0$, the statement that $f(z)$ belongs to class H_p means there exists a positive number $H_p(f)$ such that

$$\frac{1}{2\pi}\int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta < H_p(f), \qquad r < 1.$$

Let the respective Maclaurin-series expansion of $f(z)$ and $g(z)$ be $f(z) = \sum a_n z^n$ and $g(z) = \sum b_n z^n$. The statement that $h(z) = f(z) \odot g(z)$ is the Hadamard product of $f(z)$ and $g(z)$ means that $h(z) = \sum a_n b_n z^n$. A representation of the Hadamard product is the Parseval contour integral; namely,

$$h(z) = \frac{1}{2\pi i}\oint f(w) g\left(\frac{z}{w}\right)\frac{dw}{w}, \qquad |z| < |w| < 1.$$

Using the terminology of this paper, the statement of Daïovitch's theorem becomes:

Theorem 1 (Daïovitch's theorem [3]). *For $1 < p \leq 2$, if $f(z)$ belongs to class H_p and $g(z)$ belongs to class $H_q (1/p + 1/q = 1)$, then the Hadamard product $h(z)$ belongs to class B.*

Proof. Although Daïovitch proved this theorem using a method based on Poisson's integral representation of harmonic functions, a proof of this theorem may be based on Parseval's contour integral and Hölder's inequality.

The following two lemmas relate the Hardy and Lebesgue classes of functions [10; vol. 1; 150 and 277] in a manner which will be useful for this paper.

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