THE SOLUTION OF TWO SIMULTANEOUS EQUATIONS

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The purpose of this paper is to show how the methods of the approximate solution of two simultaneous equations can be reduced to the solution of one equation in one unknown.

Three methods are discussed: approximating both unknowns by use of the Newton's method for the solution of an equation in one unknown; approximating one unknown at a time by use of an iteration formula of higher order; and representing one unknown in the form of an infinite series.

The solution of three simultaneous equations in three unknowns is also discussed.

1. Let

$$f(x, y) = 0, \qquad g(x, y) = 0 \tag{1}$$

be two simultaneous equations in two unknowns x and y. Assume that one solution of each equation, (x_0, y_1) and (x_0, y_2), is known; thus $f(x_0, y_1) = 0$, $g(x_0, y_2) = 0$.

To find y_1 and y_2 for a given x_0, one can apply an appropriate method of solving an equation in one unknown. If $y_1 = y_2 = y_0$, then (x_0, y_0) is a solution of the system (1). We assume that $y_1 \neq y_2$ and that the solutions, $y = k(x)$ and $y = 1(x)$, of the two respective equations of (1), in the neighborhood of their simultaneous solution, (a, b), exist. Then the function

$$F(x) = k(x) - l(x) \tag{2}$$

is zero when $x = a$. We can approximate a by applying an iteration method to the equation $F(x) = 0$ without actually knowing $k(x)$ and $l(x)$ in an explicit form.

Differentiating equations (1) with respect to x, we obtain the equations

$$f_x(x, y) + f_y(x, y)y' = 0, \qquad g_x(x, y) + g_y(x, y)y' = 0, \tag{3}$$

which we rewrite for the points (x_0, y_1) and (x_0, y_2) as follows:

$$f_x + f_y y_1' = 0; \qquad g_x + g_y y_2' = 0, \tag{4}$$

from which

$$\begin{aligned} y_1' &= -f_x/f_y = k'(x_0) \\ y_2' &= -g_x/g_y = l'(x_0). \end{aligned} \tag{5}$$

Thus

$$F'(x_0) = k'(x_0) - l'(x_0) \quad \text{or} \quad F'(x_0) = (f_y g_x - g_y f_x)/f_y g_y \tag{6}$$

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