NORMAL, SEPARABLE MOORE SPACES AND NORMAL MOORE SPACES

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One of the best known problems in topology today is the question as to whether each normal Moore space is a metric space. Almost as interesting is the problem of proving each normal, separable Moore space is metrizable. Bing [1] and Jones [3] have results which provide much information on the problems mentioned above. One of Jones' results is that each normal, separable Moore space is metrizable, provided the continuum hypothesis is true [3, Theorem 5]. This theorem follows from another result due to Jones [3, Theorem 3], which Heath [2] recently has shown to be equivalent to the hypothesis that $2^{\aleph_0} < 2^{\aleph_1}$. As yet, efforts to remove the continuum hypothesis from Jones' result have been unsuccessful.

Neither of the above-mentioned questions is settled in this paper. Rather, this paper is devoted to establishing properties of nonmetrizable, normal Moore spaces and nonmetrizable, normal, separable Moore spaces. For instance, it is established in Theorem 3 that if there exists a nonmetrizable, normal, separable Moore space, then there exists a nonmetrizable, normal, separable, arcwise connected, locally arcwise connected Moore space. In case there exists a nonmetrizable, normal, locally compact Moore space then Theorem 7 establishes that there exists a nonmetrizable, normal, locally compact, arcwise connected, locally arcwise connected Moore space. These theorems seem to be of value since they obviously reduce the original questions. Since each Moore space is a regular, semi-metric space, it should be noted that McAuley [4, Example 3.1] has shown that regular, semi-metric spaces are not necessarily metric even if they are normal, arcwise connected, and locally arcwise connected.

A topological space is a Moore space if and only if there exists a sequence of collections of regions which satisfies Axiom 0 and the first three parts of Axiom 1 of [6]. Throughout this paper such a sequence of collections of regions is referred to as a development of the space.

A collection G is a refinement of a collection H if and only if each element of G is a subset of some element of H.

If G is a collection of point sets, then G^* denotes the point set to which x belongs if and only if x is a point of some element of G.

A collection of point sets is discrete if and only if the closures of these point sets are mutually exclusive and any subcollection of these closures has a closed sum.

A space is screenable if and only if for each open covering H of the space, there

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