AN INVOLUTION OF THE n-CELL

By Kyung Whan Kwun

1. **Introduction.** Examples of non-linear involutions of the spheres are abundant. In most cases, the non-linearity of the involution considered is based on the fact that the orbit space is not right. In their recent paper [5], Kwun and Raymond show the existence of a non-linear involution of the n-cell, for each $n \geq 5$, such that the orbit space is an n-cell. However, the fixed point set of their example is not a manifold with boundary. The purpose of this note is to prove

The Main Theorem. *There exists an involution T_n for each $n \geq 4$ of the n-cell I^n such that the orbit space is an n-cell, the fixed point set is an $(n - 1)$-cell but T_n is not linear.*

It is clear that the linearity of T_n depends entirely on the linearity of $T_n \mid \text{Bd } I^n$ if the orbit space and the fixed point set are an n-cell and an $(n - 1)$-cell respectively. In fact, our argument of the non-linearity of T_n is based on that of $T_n \mid \text{Bd } I^n$. In the above, the words the "orbit space of T_n" are an abbreviation of the orbit space of the group generated by T_n.

It should perhaps be stated that by a linear action we really mean an action equivalent to a linear one. More precisely, we say an action T on an n-cell X is linear if there exists a homeomorphism h of the unit ball B of E^n onto X and an orthogonal transformation T' of E^n such that $h^{-1}Th = T' \mid B$.

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2. **Description of T_4.** Consider I^4 and express Bd I^4 as the union of a 3-cell D and a solid horned sphere E (See [1]) along their "boundaries". Let $I_i^4 = h_i(I^4)$, $i = 1, 2$, be homeomorphic copies of I^4 under some homeomorphisms h_i.

Let X be the union of I_1^4 and I_2^4 with the sole identification that $h_1(x) = h_2(x)$ for each $x \in D$. Define T_4 as the involution of X that interchanges $h_1(x)$ and $h_2(x)$ for all $x \in I^4$. We shall show in §4 that X is a 4-cell with Bd $X = h_1(E) \cup h_2(E)$. Then it follows that

(1) T_4 is an involution of a 4-cell X,
(2) the orbit space of T_4 is a 4-cell,
(3) the fixed point set is a 3-cell and
(4) the orbit space of $T_4 \mid \text{Bd } X$ is a solid horned sphere and therefore T_4 is not linear.

3. **Induced pseudo-isotopy.** Let X be a compact Hausdorff space and G

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