CARTESIAN FACTORIZATION OF COMPACT 3- AND 4-MANIFOLDS

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1. **Introduction.** R. H. Bing [1] has proved that a topological space is a 3-cell if its cartesian product with an interval is a 4-cell. In §2 Bing's proof is generalized to show that a space is a compact 3-manifold if its product with an interval is a compact 4-manifold (by an n- *manifold* will be meant a connected separable metric space, each of whose points has a neighborhood whose closure is a topological n-cell).

A non-degenerate topological space is said to be *prime* if it cannot be expressed as a cartesian product of two non-degenerate spaces. It is proved in §3 that each compact 3-manifold which is not a cube with handles has a topologically unique factorization into prime factors. In §4 the factorization of a compact 4-manifold into 2-dimensional factors is discussed.

2. **Division of a compact 4-manifold by an interval.** Suppose that Y is a space whose cartesian product $M = Y \times I$ with the interval $I = [0, 1]$ is a compact $(n + 1)$-manifold. A sufficient number of properties of Y will be established to imply, in the case $n = 3$, that Y is a compact 3-manifold.

By the *interior* of M is meant the set of those points of M which have neighborhoods homeomorphic to Euclidean $(n + 1)$-space, and by the *boundary* of M is meant $M - \text{Int } M$. A compact manifold is called *closed* or *bounded* according as its boundary is empty or non-empty respectively. M is a bounded $(n + 1)$-manifold, because it has an interval as a factor [10], and each of the components of Bd M is a closed n-manifold.

Proposition 1. *Each of $Y \times 0$ and $Y \times 1$ is the closure of an open subset of* Bd M.

Therefore Y can be regarded as a subset of Bd M, with Bd Y and Int Y denoting the boundary and interior respectively of Y in Bd M.

Proposition 2. $\text{Bd } M = (Y \times 0) \cup (Y \times 1) \cup (\text{Bd } Y \times (0, 1))$.

Propositions 1 and 2 follow by elementary invariance of domain arguments identical to those used in the proofs of Lemmas 1 through 5 of [1]. If Bd Y is empty, then Proposition 2 implies immediately that Y is a closed n-manifold, while if Bd Y is non-empty, then Bd M is connected.

Proposition 3. Bd Y *has a finite number of components, each of which is a Peano continuum.*

Proof. By Proposition 2, Bd $Y \times (0, 1)$ is an open subset of the closed n-manifold Bd M, and is therefore locally connected. If C is a component of

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