THE GROUP G′/G″ OF A KNOT GROUP G

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1. **Algebraic theory.** The main theorems of this paper are generalizations of some results of Rapaport [6]. The title reflects my interest in the application to knot theory, which is done in §3.

Let H be an infinite cyclic multiplicative group generated by t, and denote the integral group ring by ZH. Throughout this paper A is a ZH-module which has an $n \times n$ relation matrix $M = (m_{ij}(t))$ with

$$\Delta(t) = \det M = c_d t^d + \cdots + c_1 t + c_0, \qquad c_0 c_d \neq 0.$$

The assertion that A has a relation matrix $(m_{ij}(t))$ means that there exists an exact sequence of ZH-homomorphisms

$$F_2 \xrightarrow{d_2} F_1 \xrightarrow{d_1} A \rightarrow 0, \tag{1}$$

where F_1 and F_2 are free ZH-modules with bases $(x_1, \cdots, x_n)$ and $(y_1, \cdots, y_n)$ respectively, and $d_2 y_i = \sum_{j=1}^n m_{ij}(t) x_j$, $i = 1, \cdots, n$.

Theorem (1.1). *Let R be any overring of the integers in which c_0 and c_d are units. Then $R \otimes_Z A$ is a finitely generated R-module.*

By taking for R the field Q of rational numbers, we shall obtain

Theorem (1.2) $\qquad$ rank $A = d$.

By the *rank* of A we mean the dimension of the vector space $Q \otimes_Z A$, which equals the cardinality of any maximal set of Z-linearly independent elements of A.

Denote by R_Δ the ring consisting of all $r \in Q$ such that $r = i/c_0^j c_d^k$, where $i, j, k \in Z$. Obviously R_Δ is the smallest ring lying between Z and Q in which c_0 and c_d are units.

Theorem (1.3). *If* g.c.d. $(c_0, \cdots, c_d) = 1$, *then A is a torsion free Z-module.*

In the remainder of this section it is assumed that R is an arbitrary ring for which $Z \subset R \subset Q$ and that A, as a Z-module, is torsion free.

Theorem (1.4). *If $R \otimes_Z A$ is a finitely generated R-module, then c_0 and c_d are units of R* (i.e., $R_\Delta \subset R$) *and*

$$R \otimes_Z A = \bigoplus_{i=1}^{d} R.$$

Consider the map $\varphi : A \rightarrow R \otimes_Z A$ given by $\varphi a = 1 \otimes a$. When is φ an isomorphism and $R \otimes_Z A$ finitely generated? A necessary condition is that

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