A SEQUENCE OF RATIONAL FUNCTIONS WITH APPLICATION TO APPROXIMATION BY BOUNDED ANALYTIC FUNCTIONS

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A series of interpolation in the z-plane whose terms are rational functions and which can be used for the expansion of an arbitrary function analytic on a suitable point set, was recently described by the present writer [5]; that series is closely connected with a canonical domain for the mapping of an arbitrary multiply connected region [6], and the series has applications [5], [7] to approximation by bounded analytic functions. The present paper describes a new series of rational functions that is no longer a series of interpolation but nevertheless shares the more important delicate convergence properties of the previous series; the new series is related to a conformal map treated by H. J. Landau [3], and likewise has applications (Theorems 3 and 4, below) to approximation by bounded analytic functions. The main advantage of the new series is that it can be used in the study of various kinds of approximation on *non-analytic* Jordan curves, in contrast to the older series.

A special case of the new series involves approximation by polynomials on a set of mutually exterior Jordan curves that need not be analytic; this subject was treated by Walsh and Sewell [10] in a paper whose details seem to require some modification, as we discuss below. To be explicit, in that paper points of interpolation were distributed equally on the set of Jordan curves with respect to a suitable parameter; in the present paper those points are distributed equally on *each* of the Jordan curves with respect to that parameter.

We say that a Jordan curve is *of class A* provided it can be represented parametrically in terms of arc length s by $x = x(s)$, $y = y(s)$, where $x(s)$ and $y(s)$ possess second derivatives with respect to s which satisfy a Lipschitz condition of some positive order in s. Such a curve remains of class A under one-to-one conformal transformation of a region containing it; compare Warschawski [11; 450].

THEOREM 1. *Let D be a region of the extended z-plane whose boundary consists of mutually exterior Jordan curves $B_1, B_2, \cdots, B_\mu$ each of class A, together with the finite points $a_1, a_2, \cdots, a_{\nu-1}$ ($\nu \geq 1$) each exterior to each B_j and the point at infinity a_ν. Suppose $\Sigma_1^\nu n_j = 1$, $n_j > 0$, and let $u(z)$ be the unique function harmonic in D, continuous and equal to zero on $B = \Sigma_1^\mu B_j$, and in the neighborhood of each finite a_j of the form $u_j(z) - n_j \log |z - a_j|$, where $u_j(z)$ is harmonic at a_j, and in the neighborhood of $z = \infty$ $u(z)$ is of the form $u_\nu(z) + n_\nu \log |z|$, where $u_\nu(z)$ is harmonic at infinity.*

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