MULTIPLE TRIGONOMETRIC SERIES OF A PARTICULAR TYPE

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1. **Introduction.** The problems of uniqueness, representation, and localization of multiple trigonometric and Fourier series have been considered by Bochner [2], Cheng [5], Herriot [7], [8] Berkovitz [1], Shapiro [9], and others. Bochner [2] proved localization properties for multiple Fourier series where the summability method is Toeplitz summation of "spherical" partial sums. Cheng [5] used (C, 1) summation of spherical partial sums to obtain a uniqueness theorem for multiple trigonometric series. Under a generalized Abel summation of multiple trigonometric series, Shapiro [12] obtained a representation of the coefficients in terms of the Lebesque integral, and under convergence and (C, 1) summability of circular partial sums of double trigonometric series he [10], [11] obtained standard type uniqueness theorems. Herriot [7], [8] considered localization properties of multiple Fourier series when the summation method is Nörlund summation of rectangular and "triangular" partial sums. He has shown, for example, that a regular Nörlund summation of triangular partial sums has the, localization property for double Fourier series but not for series of higher dimension. Shapiro [9] and Berkovitz [1] have studied localization properties for double trigonometric series, not known to be Fourier series; Shapiro's result is stated in terms of square summation, Berkovitz's in terms of circular summation, of the series.

In this paper properties of a multiple trigonometric series of a special type are investigated: it is shown that for convergence of the triangular partial sums, reasonable uniqueness and localization theorems hold; and Theorem 4.3 gives a uniqueness result in a form which is perhaps unexpected, although in one dimension it reduces to the standard result that a trigonometrical series converging everywhere to 0 vanishes identically. The same methods will yield proofs of Theorems 4.2 and 4.3 with triangular convergence replaced by triangular summability (C, k). Under these conditions the inner integral in (4.8) becomes a $C_{k+1}P$ – integral.

2. **Definitions.** Let

$$\sum_{0}^{\infty} \cdots \sum_{0}^{\infty} a_m \cos mx + b_m \sin mx \tag{2.1}$$

be a trigonometric series in n variables where $m = m_1 m_2 \cdots m_n$, $mx = m_1x_1 + m_2x_2 + \cdots + m_nx_n$ and a_m, b_m are real for all m. (This series is a special case of series (1.1) [5] where $f_{mn} = a_{mn}$, $d_{mn} = b_{mn}$.) The series

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