ON THE CONTINUATION OF SOLUTIONS OF THE EQUATIONS OF ELASTICITY BY REFLECTION

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1. **Introduction.** A number of continuation formulas for solutions of the equations of elasticity may be found in the literature. Duffin [5] has considered the problem of continuing a solution across a plane boundary Q when either the displacements or surface tractions vanish on Q. Bramble [2], [3] has treated the analogous problem for Q spherical. He has also derived a continuation formula [3] for the case in which a linear combination of the displacements and surface tractions vanish on the spherical surface Q. More recently the authors [4] demonstrated that it was possible to obtain an explicit representation for the continuation of the solution across a spherical surface Q if on Q either (i) the normal components of displacement, surface traction, and rotation vanish or (ii) the tangential components of displacement and surface traction vanish.

In this paper we demonstrate how by reflection it is possible to continue the solution of the equations of elasticity across a spherical surface Q when a linear combination of the normal components of displacement and surface traction and a linear combination of the tangential components of displacement and surface traction vanish on Q. Special cases of these conditions yield results for all of the boundary values normally encountered in physical situations.

2. **Continuation of the displacement vector.**

Let u_i satisfy

$$\Delta u_i + \alpha u_{j,ji} = 0, \qquad i = 1, 2, 3, \qquad \alpha = (\lambda + \mu)/\mu \tag{1}$$

in a three-dimensional region D, a portion of whose boundary Q is an open subset of $r = a$. The symbol Δ denotes the Laplace operator and summation from 1 to 3 is implied when an index is repeated. In the usual manner a comma indicates partial differentiation. Let p be a point of D and let p' be the inverse point with respect to the surface $r = a$. Denote by D' the reflection of D in in $r = a$ and let $D^* = D \cup D' \cup Q$. If D^* is such that for all points p which belong to D, the line joining p and p' is completely contained in D^*, then for $\alpha \neq -1$, u_i may be continued as a solution of (1) into D^* provided that on Q,

$$a_1 \frac{1}{\lambda} x_i x_j \tau_{ij} + b_1 x_j u_j = 0 \tag{2}$$

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