A CONTINUOUS POKER GAME

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1. Introduction. In this paper we derive the solutions of a zero-sum two-person poker game in which the players' hands are independent random numbers from the interval [0, 1]. The game involves two bet levels a, b and an ante of 1 unit ($a > b > 1$). The players act alternately, and one of them is permitted a single raise.

Our model subsumes the alternating-bid von Neumann poker game of [4] as well as the model solved by Bellman [1]. The former arises from the limiting case $b = 1$, the latter as the "equal increments" case $a - b = b - 1$. (Karlin and Restrepo [3] have recently solved the equal increments game with n rounds of bidding.) The solution exhibits qualitative features like those of [4] and [1] and turns out to depend on a decomposition of [0, 1] into three subintervals corresponding to low hands, intermediate hands and high hands. Optimal strategies for the players are distinguished among the *semioptimal* strategies (those which achieve the value of the game against every optimal strategy of the opponent) by a specification of the average frequency of bluffing over the range of low hands and (for one player) by an integral sublinearity condition on the frequency of seeing a raise when holding an intermediate hand.

The analysis of a symmetric version of the model has also been carried out; the results will appear elsewhere [2].

2. Description of the game and its solution. The rules of the game are as follows. The two players R and S first *ante* 1 unit. They then receive independent random numbers (*hands*) from the interval [0, 1]. Each player knows his own hand, but not that of his opponent. Player S is first to act; he either *drops* (in which case play ends) or bets an additional $b - 1$ units. If play continues, then Player R can either drop (in which case play ends), *see* by also betting an additional $b - 1$ units (in which case play ends), or *raise* by betting and additional $a - 1$ units. If play continues, then Player S has the option of either *folding*, or seeing by betting an additional $a - b$ units. Naturally we assume $a > b > 1$. A player who drops or folds loses his ante and previous bets (if any) to his opponent. After a "see" the player with the higher hand wins the ante and previous bets (if any) of his opponent. No payment occurs if the hands are equal. This event has probability zero, and so can be ignored.

Each player has three courses of action for any given hand. For Player S, these are (1) to drop, (2) to bet $b - 1$ units with the intention of folding if raised, and (3) to bet $b - 1$ units with the intention of seeing if raised. The courses of

Received January 31, 1958; in revised form, August 10, 1959. The second author is now at Stanford University.